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DENSITY AND MECHANICAL PROPERTIES MATHEMATICAL MODELING OF COMPOSITE SANDWICH STRUCTURAL OPEN CELL FOAM

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ABSTRACT

Light weight Open cell polymer foam composites sandwich structures are being introduced for the use in applications of aerospace structures to automobile components. A mathematical model for foam density developed by using box and tetrahedron methods showed 90% accuracy when validated with the experimental results. The research may be extended to the characterization of thermal properties for wider applicability of the developed sandwich structures based on open cell polymer foam cores. In this paper, work has been analyzed composite sandwich structural foam and their density modeling method in mathematical approach.

Keywords: *Polymer composite, Sandwich structure, Foam density, Epoxy foam, Polyester foam.*

1. Introduction

The use of composite sandwich structural Open/Closed cell foams, applications in aeronautical, automotive, aerospace, marine engineering. These sandwich structures have an excellent stiffness-to-weight ratio that leads to weight reduction, fuel consumption and high structural crashworthiness capable of absorbing high amount of energy in a sudden impact. Different combination of Epoxy/Polyester core and face sheet materials are being studied and experimenting by academic researchers across the world in order to achieve better crashworthiness [1-2].Polymer composite Open/Closed cell foams are developed in a number of different methods by slab-stock by pouring, extrusion and disparate forms of mold. The reasons for Open/Closed cell polymer foams are extensively used because of numerous beneficial properties like density is low; mass reduction compared to additional metal matrix composites option is major [3].To derive mathematical models for forecast, assumed Open/Closed cell foam structure, fundamental material constitutive laws and beginner's elasticity theories need to be considered to ensure any form of Experimental/Numerical/Mathematical accomplishment.

Clear-cut cubic Open/Closed cell foam assuming material isotropy is a wide choice of elastic, plastic, and damage semi-empirical models which is under tension, compression and shear experimented [6]. Prominent foam model stands nowadays until it is the solitary space-filling polyhedron with the least amount surface-area-to-volume ratio; next after decades, managed to beat the speculation with an array of six 14-faces polyhedral and two 12-faces polyhedral. Still with these landmark achievement, environment continues to confront research scientists to recommend models to simulate complex cellular morphology of polymeric Open/Closed cell foams today [6, 8-9].The mechanics of foams have been investigated under the tensile deformation of open-cell foams. Analyzing both Open/Closed cell foam shows that the deformation mechanisms [7]. Low-velocity impact response of composite Open/Closed cell foam structures is the foundation for prediction of residual strength (tension and compression) after impact [10]. A measure of advance has been made in the study of Open/Closed cell foam sandwich structures and a worldwide realization that uncomplicated structural theories, such as, link, classic beam or plate theory are not ample to understand

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stress and strain distributions, under moderately concentrated loads [11].

2. Density Model

2.1Box method of calculation

Box modeling is a very basic concept. Each face would be divided up into smaller or more detailed faces. This basic box modeling shape in open cell foam structural 2 which is about refinement and redefining used to final model to particular main areas of applications ranging from domestic house hold to Industries in aircraft, spacecraft, submarine, ships and boats, surface transport vehicles, building materials, packaging materials, thermal and electrical insulation, storage tanks etc. To reasonable success, simple cubic foam cells or into specific geometrical shapes (Box Type or Tetrahedral type) assuming material isotropy, a wide range of elastic, plastic, and damage semi-empirical models for open-cell foams structures, mathematical models derive for forecast open cell foam structures, considering basic material constitutive laws, and simplified elasticity theories. Mechanical classification of foams has its fair share of methodological and methodical challenges.

Consider a box of volume a x a x a (a^3) for preparation of polymer foams, spheres of diameter's' filled in this mould shown in figure1. The 'n' number of spheres filled in the box is given

$$n = \frac{a^3}{d^3} \qquad Eq.2.1$$

The volume of resin (V_r) filled gaps between the balls is given = volume of box – n x volume of sphere

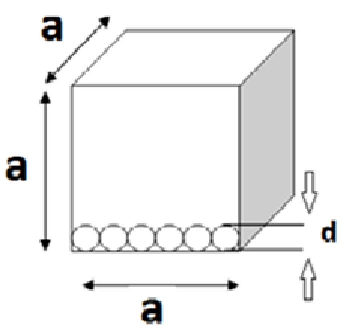

Figure 1Box Method model

$$v_r = a^3 - n\frac{4}{3}\pi r^3 \qquad Eq.2.2$$

$$v_r = a^3 - \frac{a^3}{d^3}\frac{4}{3}\pi r^3 \qquad Eq.2.3$$

$$v_r = a^3 - \frac{a^3}{d^3}\frac{4}{3}\pi\frac{d^3}{8} \qquad Eq.2.4$$

$$= a^3\left[1 - \frac{\pi}{6}\right] \qquad Eq.2.5$$

$$v_r = 0.476\, a^3 \qquad Eq.2.6$$

Mass of the resin can be calculated

$$M_r = v_r \,x\, \partial_r = 0.476\, \partial_r a^3 \qquad Eq.2.7$$

If there is no porosity in the casting then the mass of the resin required (M_r) is = volume of box x density of resin epoxy/polyester

$$M_r = a^3 x\, \partial_{e/p} \qquad Eq.2.8$$

After removing spheres from the casting, the obtained volume of porosity is given by

= n x volume of balls

$$= n\frac{4}{3}\pi r^3 \qquad Eq.2.9$$

The percentage of porosity can be calculated

= volume of porosity/Total volume *100

$$= \left[\frac{a^3 - 0.476\, \partial_r a^3}{a^3}\right] * 100 \qquad Eq.2.10$$

$$[1 - 0.476] * 100 \qquad Eq.2.11$$

$$52.3\%$$

Then calculating density of foam (δ_f) for polyester/epoxy = Volume of resin x density of polyester/epoxy

$$d_f = v_r \,x\, \partial_p \text{ or } v_c \,x\, \partial_e \qquad Eq.2.12$$

$$d_f = 0.476\, a^3 x\, \partial_{p/e} \qquad Eq.2.13$$

The density of open cell for any material can be calculated by using above equation.

2.2 Tetrahedral method.

Tetrahedral modeling is a very basic concept where each face would be divided up into smaller or more detailed faces. This basic box modeling shape in open cell foam structural 2 which is about refinement and redefining used to final model to particular main areas of applications ranging from domestic house hold to Industries in aircraft, spacecraft, submarine, ships and boats, surface transport vehicles, building materials, packaging materials, thermal and electrical insulation, storage tanks etc. To reasonable success, simple cubic foam cells or into specific geometrical shapes (Box Type or Tetrahedral type) assuming material isotropy, a wide range of elastic, plastic, and damage semi-empirical models for open-cell foams structures, mathematical models derive for forecast open cell foam structures, considering basic material constitutive laws, and simplified elasticity theories. Mechanical classification of foams has its fair share of methodological and methodical challenges.

Consider a box of volume a x a x a (a^3) for preparation of polymer foams, spheres of diameter's 'filled in this mould in tetrahedral shape shown in figure 2. The 'n' number of spheres filled in the box is given by Pythagorean Theorem

$$h^2 = [2r^2 - r^2] \qquad \text{Eq. 2.21}$$

$$h = \frac{r}{\sqrt{3}} \qquad \text{Eq. 2.22}$$

Number of balls/unit length of a cube

$$n = \quad n = \frac{2}{\sqrt{3}}\frac{a^3}{d^3} \qquad \text{Eq. 2.23} \quad \text{,if } r = d/2$$

The volume of resin (Vr) filled gaps between the balls triangularly is given = volume of box – n x volume of sphere

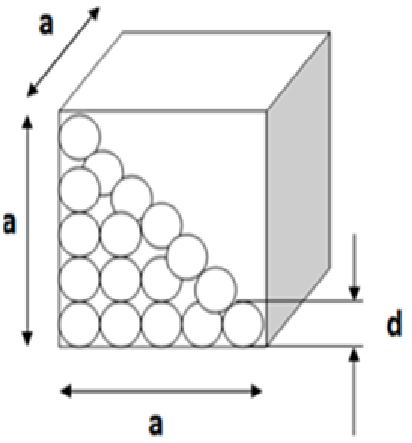

Figure 2 Tetrahedral Model

$$v_r = a^3 - \frac{2}{\sqrt{3}}\frac{a^3}{d^3}\frac{4}{3}\pi r^3 \qquad \text{Eq. 2.24}$$

$$= a^3 - \frac{2}{\sqrt{3}}a^3\frac{\pi}{6} \qquad Eq. 2.25$$

$$a^3[1 - 0.6048] \qquad \text{Eq. 2.26}$$

$$v_r = 0.395\ a^3 \qquad \text{Eq. 2.27}$$

Mass of the resin can be calculated

$$M_r = v_r \text{ x } \partial_r = 0.395\ \partial_r a^3 \quad \text{Eq. 2.28}$$

If there is no porosity in the casting then the mass of the resin required (M_r) is = volume of box x density of resin epoxy/polyester

$$M_r = a^3 \text{x} \partial_{e/p} \qquad \text{Eq. 2.29}$$

Density of Polyester ∂_p / Density of Epoxy ∂_e

After removing spheres from the casting, the obtained volume of porosity is given by

= n x volume of balls

$$= n\frac{4}{3}\pi r^3 \qquad Eq. 2.30$$

The percentage of porosity can be calculated = volume of porosity/Total volume *100

$$\left[\frac{a^3 - 0.395\ \partial_r a^3}{a^3}\right] * 100 \qquad \text{Eq. 2.31}$$

$$[1 - 0.395] * 100 \qquad Eq. 2.32$$

$$60.48\%$$

Then calculating density of foam (δ_f) for polyester/epoxy = Volume of resin x density of polyester/epoxy

$$d_f = v_r \text{ x } \partial_p \text{ or } v_c \text{ x } \partial_e \qquad \text{Eq. 2.33}$$

$$d_f = 0.395\ a^3 \text{x } \partial_{\frac{p}{e}} \qquad \text{Eq. 2.34}$$

The experimental density should be between rectangle method and tetrahedral method. % of decrease in density= 0.395

3. Mechanical properties

3.1 Box method of calculation

Box modeling is a very basic Concept each face would be divided up into smaller or more detailed faces. This basic box modeling shape in open cell foam structural is about refinement, and redefining used to final model.

Consider a section area of a square a x a (a^2) for prepared foam. The 'n' number of spheres filled in the box is given by

$n = \frac{a^2}{d^2}$ $\quad Eq.3.1$ shown in figure 3

Where "d"= diameter of spheres.

Solid area = Total area of a square – n x area of sphere

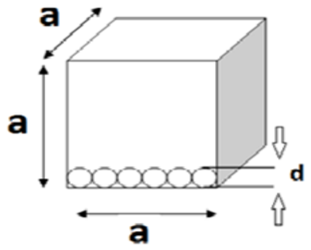

Figure 3Box Method model

$$A = a^2 - \frac{a^2}{d^2}\pi\frac{d^2}{4} \qquad Eq.3.2$$

$$a^2 - \frac{\pi}{4}a^2 \qquad Eq.3.3$$

$$A = a^2\left[1 - \frac{\pi}{4}\right] \qquad Eq.3.4$$

Stress is a measure of external force applied on the solid structures. Internal resisting force opposes external force per unit area of a surface of the body is given by Force/Area.

The tensile strength of the foam is given by

$$\sigma_f = \frac{p}{a^2}\left[1 - \frac{\pi}{4}\right] \qquad Eq.3.5$$

Tensile strength of the solid specimen

$$\sigma_s = \frac{p}{a^2} \qquad Eq.3.6$$

Ratio of reduction of tensile strength is given by $= \frac{\sigma_f}{\sigma_s} * 100 \quad Eq.3.7$

$$= \left[\frac{\frac{p}{a^2}\left[1 - \frac{\pi}{4}\right]}{\frac{p}{a^2}}\right] * 100 \qquad Eq.3.8$$

42%

Specific strength reduction = $\left[\frac{\frac{\sigma_f}{\sigma_s}}{\frac{\sigma_f}{\sigma_s}}\right] \quad Eq.3.9$

$$\left[\frac{\frac{p}{a^2}\left[1 - \frac{\pi}{4}\right]}{\frac{p}{a^2}}\right] * \left[\frac{0.476\,\partial_p a^3}{a^3\,\partial_p}\right] \qquad Eq.3.10$$

0.19%

3.2 Tetrahedral method

Tetrahedral modeling is a very basic Concept each face would be divided up into smaller or more detailed faces. This basic box modeling shape in open cell foam structural is about refinement, and redefining used to final model.

Consider a solid area of a square a x a (a^2) for prepared foam the 'n' number of spheres filled in the box is given by n = $n = \frac{2}{\sqrt{3}}\frac{a^2}{d^2} \quad Eq.3.21$ Shown in figure 4.

Where "d"= diameter of spheres.

Solid area = Total area of a square – n x area of sphere

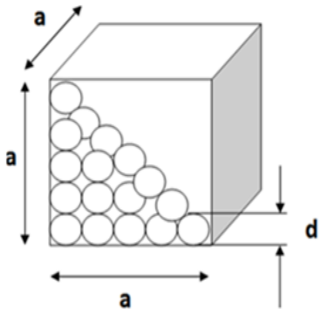

Figure 4 Tetrahedral Model

$$A = a^2 - \frac{2}{\sqrt{3}}\frac{a^2}{d^2}\pi\frac{d^2}{4} \qquad Eq.3.22$$

$$a^2 - 0.578\frac{\pi}{4}a^2 \qquad Eq.3.23$$

$$A = a^2[1 - 0.453] \qquad Eq.3.24$$

Stress is a measure of external force applied on the solid structures. Internal resisting force opposes external force per unit area of a surface of the body is given by Force/Area.

The tensile strength of the foam is given by

$$\sigma_f = \frac{p}{a^2}[1 - 0.453] \qquad Eq.3.25$$

Tensile strength of the solid specimen

$$\sigma_s = \frac{p}{a^2} \qquad Eq.3.26$$

Ratio of reduction of tensile strength is given by $= \frac{\sigma_f}{\sigma_s} * 100 \quad Eq.3.27$

$$= \left[\frac{\frac{p}{a^2}[1 - 0.453]}{\frac{p}{a^2}}\right] * 100 \qquad Eq.3.28$$

$1.82 * 100\ \%$

Specific strength

$$\text{reduction} = \left[\frac{\frac{\sigma_f}{\sigma_s}}{\frac{\sigma_f}{\sigma_s}}\right] \qquad Eq.3.29$$

$$\left[\frac{\frac{p}{a^2}[1 - 0.453]}{\frac{p}{a^2}}\right] * \left[\frac{0.395\ \partial_p a^3}{a^3 \partial_p}\right] \qquad Eq.3.30$$

$1.82 * 0.395\ \%$

0.718

4. Density measurement experimentally

4.1 Procedure to measure foam

A wooden mould was prepared of dimension of 150mm× 150mm×10mm inside the cavity of mould was greased to avoid polymer sticking into the mould. Six millimeter sugar balls are filled in the cavity with little vibration. Calculated amount of polyester resin (106g) is taken in a jar and hardener 2% of accelerator (methyl ethyl ketone peroxide) and 2% of catalyst (diethyl acetamide) is added into the jar and stirred for 10 min and poured into the cavity. 24 hours for solidification / curing was allowed. The sugar balls, placed in the hot water for 24 hours are removed. The similar steps are taken for epoxy resin with 10% amine based hardener.

Prepared open cell polymer density is measured as per ASTM D792-00 standard. A wooden mould was prepared of dimensions into box shape of size 150 mm x 150 mm x 10 mm. measuring the length, width and depth of the specimen of foam by using screw gauge of an accuracy of 0.01mm and calculate the volume of the foam is calculated. The weight of foam specimen is measured by using electronic balance (0.01 mg accuracy).

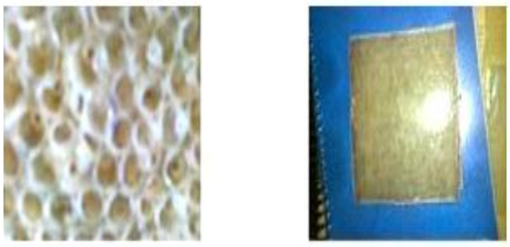

Figure 5 Photos of the foam.

The density of foam is calculated using formula.

$$\rho = \frac{m}{v} \qquad Eq.4.1$$

Where ρ is the density of foam in g/cm^3, m is the mass grams, and v is the volume of foam specimen cm^3. The six specimens were measured for each condition.

5. Comparison of density model between mathematical model and experimental method calculation

5.1 Box method density model

Table 1: Comparison of Box method density model between mathematical model and experimental method calculation

Type of foam material	Mathematical calculation for density model	Experimental calculation for density model	% error in density model
Epoxy foam	0.66	0.84	21%
Polyester foam	0.57	0.76	25%

In the above Table1, two types of foam are considered one is epoxy foam and other is polyester foam and density is calculated mathematically and the results obtained are for epoxy foam 0.66 and polyester foam 0.57 and experimentally prepared polymer open cell foam density is calculated and the results are for epoxy foam 0.84 and for polyester foam 0.76 and their percentage reduction in density model between mathematical model and experimental method is 21.45% for epoxy foam and 25% reduction in polyester foam.

5.2 Tetrahedral density model

Table 2: Comparison of Tetrahedral density model between mathematical model and experimental method calculation

Type of foam material	Mathematical calculation for density model	Experimental calculation for density model	Eerror in density model
poxy foam	0.55	0.84	34%
olyester foam	0.47	0.76	38%

In the above Table 2, two types of foam one is epoxy foam and other is polyester foam are considered and density is calculated mathematically. The results obtained are for epoxy foam 0.55 and polyester foam 0.47. The experimentally prepared polymer open cell foam density is calculated and the results for epoxy foam is 0.84 and for polyester foam is 0.76.Their percentage reduction in density model between mathematical model and experimental method is 34% for epoxy foam and 38% reduction in polyester foam. The polyester foam shows higher error compared to epoxy foam.

6. Conclusion

- Two types of foam are considered, one is epoxy foam and other is polyester foam and density is calculated mathematically.
- In the above Table 1(A) Box density model the results obtained are for epoxy foam 0.66 and polyester foam 0.57 and experimentally prepared polymer open cell foam density is calculated and the results; are for epoxy foam 0.84 and for polyester foam 0.76 between mathematical model and experimental method.
- Their percentage reduction in density model between mathematical model and experimental method is 21.45% for epoxy foam and 25% reduction in polyester foam.
- In the above Table 1(B) Tetrahedral density model polyester foam the density is calculated mathematically and the results obtained for epoxy foam is 0.55 and polyester foam is 0.47.
- In the experimentally prepared model polymer open cell foam density is calculated and the results for epoxy foam is 0.84 and for polyester foam is 0.76 between mathematical model and experimental method.
- Their percentage reduction in density model between mathematical model and experimental method is 34% for epoxy foam and 38% reduction in polyester foam.
- The polyester foam shows higher error compared to epoxy foam.

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