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THREE DIMENSIONAL ANALYSIS OF RECTANGULAR PLATE WITH HOLE UNDER IN PLANE LOADING CONDITIONS FOR BOTH ISOTROPIC AND ORTHOTROPIC MATERIAL UNDER VARYING PARAMETRIC CONDITIONS

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ABSTRACT

In this work, in-plane stress concentration factor (SCF)for a rectangular isotropic plate with a hole under in plane loading conditions is investigated. The influence of the various geometric parameters like diameter-to-width (D/A) and thickness-to-width (T/A) ratios on SCF is carried out. SCF for various stresses along x, y and z direction including von mises stresses is being evaluated under varying parametric conditions. All results are presented in graphical form and discussed. The work is further extended for five different composite materials also, so that variation of SCF for varying elastic constants can easily be studied.Based on the result, it is concluded that in-plane loading conditions remains no morea two dimensional problem instead it's a three dimensional problem.

Keywords: *Finite Element Method, Stress Concentration Factor, Composites, Transverse Loading,Deflection and Elastic Constants*

1. Introduction

Stress concentration arises from any abrupt change in geometry of the plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occurs at points of stress concentration. Hence, for the design of a rectangular plate with a central circular hole, stress concentration factor plays an important role and thus accurate knowledge of stresses and stress concentration factor at the edges of the hole under in the plane or transverse loading are required. Manufacturing of components along with discontinuities requires the prediction of stress concentration analysis for both isotropic and anisotropic materials. This leads to usage of different type of materials based on their application and strength requirement.

Paul and Rao [1,2]presented a theory for evaluation of stress concentration factor of thick and FRP laminated plate with the help of Lo-Christensen-Wu higher order bending theory under transverse loading. Shastry and Raj [3] have analyzed the effect of fibre orientation for a unidirectional composite laminate with finite element method by assuming a plane stress problem under in plane static loading. Xiwu et al. [4, 5] evaluated stress concentration of finite composite laminates with elliptical hole and multiple elliptical holes based on classical laminated plate theory. Iwaki [6] worked on stress concentrations in a plate with two unequal circular holes. Ukadgaonker and Rao[7] proposed a general solution for stresses around holes in symmetric laminates by introducing a general form of mapping function and an arbitrary biaxial loading condition in to the boundary conditions. Ting et al. [8] presented a theory for stress analysis by using rhombic array of alternating method for multiple circular holes. Chaudhuri [9]worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson [10]has developed good theory and charts on the basis of mathematical analysis and presented excellent methodology in graphical form for evaluation of stress concentration factors in isotropic plates under in-plane loading with different types of abrupt change, but no results are presented for transverse loading.Troyani et al. [11] have determined the in-plane theoretical stressconcentration factors for short rectangular plates with centered circular holes subjected to uniform tension using finite element method. Kotousov and Wang [12] have presented analytical solutions for the three dimensional stress distributions around typical stress concentrators in an isotropic plate of arbitrary thickness based on the assumption of a generalized plane strain theory. Toubal

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et al. [13] studied stress concentration in a circular hole in composite plate. Ukadgaonker and Kakhandki [14] analyzed the stress around an irregular shaped hole for different in-plane loading conditions for an orthotropic fibrous plate. Gruber et al. [15] developed analytical solution methods for the analysis of stress concentration in fibre reinforced multilayered composites with pin loaded holes. She and Guo [16] have analyzed the variation of three dimensional stress concentration factors along the wall of elliptic holes in finite thickness plates of isotropic materials subjected to remote tensile stress using finite element method. Jain and Mittal [17] have analyzed the stress concentration and deflection in isotropic, orthotropic and laminated composite plates with central circular hole subjected to transverse static loading by using two dimensional finite element methods. Mittal and Jain [18] analyzed the effect of fibre orientation on stress concentration factor in fibrous plate with a central circular hole under transverse static loading by using two dimension finite element methods. Ghezzo et al. [19] performed a numerical and experimental analysis of the interaction between two notches in carbon fibre laminates. The numerical analysis of the stress distribution in-plane stress assumption and within the fibrous plate theory framework has been conducted on two symmetric laminates. Ozben et al. [20] compiled FEM analysis of laminated composite plate with rectangular hole and various elastic modulus under transverse loads.. Yu et al. [21] have performed the measurement of the m-integral for a hole in an aluminumplate or strip.A method using Digital Image Correlation is proposed to measure the M-integral in an elastic rectangularplate and elastic–plastic strip made of LY12 aluminumwhere a hole is located at center as a defect.It is concluded that the M-integral for an elasticrectangular plate with a hole made of LY12 aluminumshows a clear path-independent property. Zappalorto et al. [22] have done in-plane and out-of-plane stress field solutionsfor v-notches with end holes. The aim of this research is to provide a solution for V-notches with end hole and varying opening angles with two different loading conditions.Results for in-plane problem has been addressed by means of theKolosov-Muskhelishvili approach, using two potentialfunctions whose exponents were simple combinationsof Williams' eigenvalues for pointed V-notches.

In view of the above review, it can be concluded that the analysis of in-plane stress concentration in isotropic and composite plates with hole subjected to in-plane loadings needs to be further investigated in detail.The present study is concern with effect of the geometric and material parameters on the stresses along different direction.Variation of SCF is being observed for both isotropic and composite materials.

1.1 Formulation of the problem

The model of rectangular plate of dimension 0.2 *m* X 0.1 *m*and*T* thicknesswith a central circular hole of diameter *D* under in-plane loading condition (Fig. 1.) is taken for analysis. Three different *T/A* ratios of 0.01, 0.05 and 0.10 are varied and for each *T/A* ratio, *D/A* ratio is being varied from 0.1 to 0.7 respectively. In-plane loading conditions is being provided by keeping one edge fixed and a uniform axial loading of 1 N/m^2 is applied over the other end of the plate. The material properties for the six different materials selected are shown in table 1.

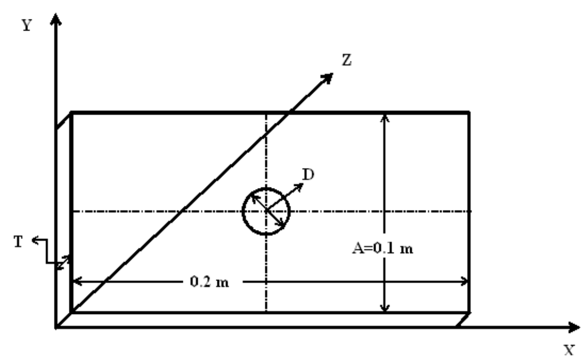

Fig. 1 Rectangular Plate With Hole

1.2 Finite element analysis

Finite element method was chosen for analysis of the model. The model was meshed using a 3-D solid element, Solid 186 with three degrees of freedom and 60 nodes per element in ANSYS. Mapped meshing is used, so that more elements employed near the hole boundary. Element length is selected as 0.002 m after running the convergence tests. The example of the discretizedthree dimensional finite element model and geometry of plate, used in the study is shown in Figure 2.

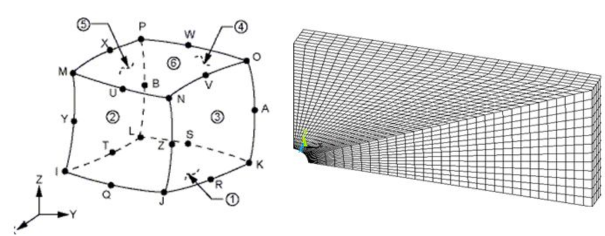

Fig. 2 Element used and Generated Mesh of the Plate

Table 1:Material Properties of Composite Materials [23]

Properties \ Material	Isotropic	E-glass/ Epoxy	Boron/ Epoxy	Woven-Glass/Epoxy	Boron/ Aluminum	Graphite/ Epoxy
E_x	39GPa	39GPa	201GPa	29.7GPa	235GPa	294GPa
E_y	-	8.6GPa	21.7GPa	29.7GPa	137GPa	6.4GPa
E_z	-	8.6GPa	21.7GPa	29.7GPa	137GPa	6.4GPa
G_{xy}	-	3.8GPa	5.4GPa	5.3GPa	47GPa	4.9GPa
G_{yz}	-	3.8GPa	5.4GPa	5.3GPa	47GPa	4.9GPa
G_{zx}	-	3.8GPa	5.4GPa	5.3GPa	47GPa	4.9GPa
μ_{xy}	0.30	0.28	0.17	0.17	0.30	0.23
μ_{yz}	-	0.28	0.17	0.17	0.30	0.23
μ_{zx}	-	0.28	0.17	0.17	0.30	0.23

Here, *E*, *G* and μ represent modulus of elasticity, modulus of rigidity and Poisson's ratio.

2. Result and Discussion

In the present case, variation of SCF for stresses along x, y, z and von-misses stress (σ_x, σ_y, σ_z and σ_{eqv}) versus D/A ratio for six different materials are shown in the figures below. Results are discussed case by case further.

Figure. 3 shows the variationof SCF (σ_x)versus D/A ratio for different materials considered. All the materials shown in figure followa similar pattern but the values of SCF are different for all the materials. The maximum SCF value obtained for isotropic, e-glass/epoxy, woven-glass/epoxy, boron/epoxy, graphite/epoxy and boron/aluminum are 2.73, 5.74, 4.72, 7.34, 6.63 and 4.37 at D/A=0.1 and T/A=0.10 respectively, whereas minimum SCF obtained are 2.11, 2.31, 2.28, 2.42, 2.30 and 2.25 at D/A=0.7 and T/A=0.01 respectively. Thus the maximum SCF value is obtained for boron/epoxy and minimum for isotropic material. From the graphs it can be clearly seen that variation of D/A is showing significant effects as an increasing the D/A ratio from 0.1 to 0.7, SCF value continuously decrease for all the materials correspondingly, whereas the very less effect is observed for varying T/A ratio.

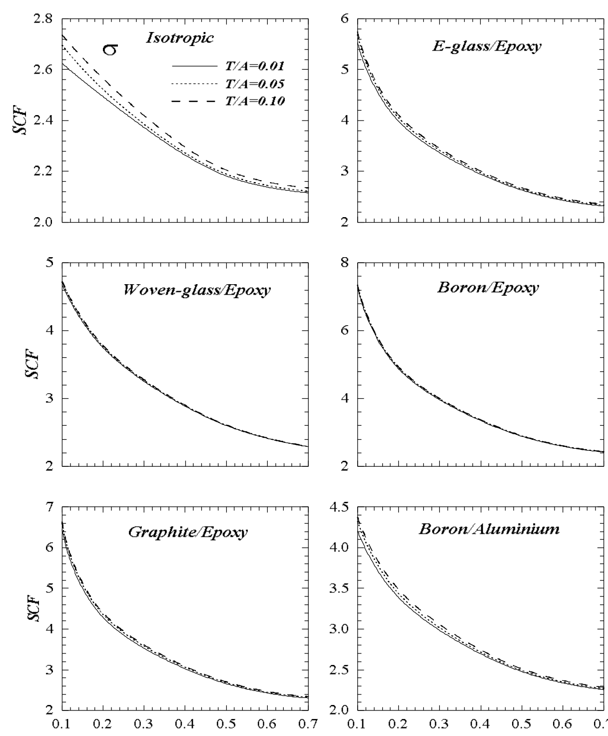

Fig. 3 Variations of SCF for(σ_x) versus D/A ratio

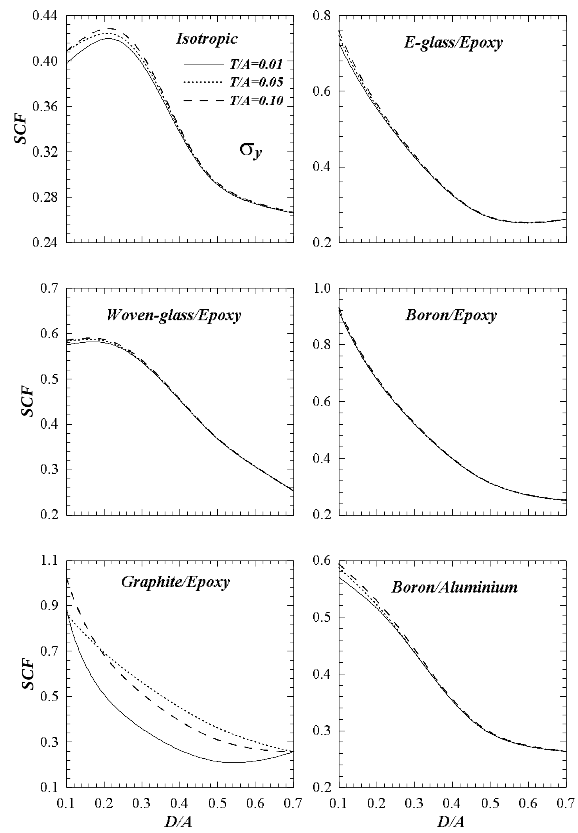

Fig. 4 Variations of SCF for(σ_y) versus D/A ratio

Figure. 4 shows the variationof SCF (σ_y)versus D/A ratio for different materials considered. From the graphs it can be clearly observed that isotropic, woven-glass/epoxy and boron/aluminum follow the similar trend for increasing values of SCF with a corresponding increase in the D/A ratio up to 0.2 and then decrease in SCF with a subsequent increase in D/A ratio. E-glass/epoxy, boron/epoxy and graphite/epoxy follows a similar pattern for the continuous decrease in SCF with a corresponding increase in D/A ratio. The dissimilarity between the pattern of the materials is due to Ex/Ey and Ex/Gxy ratios of materials. Isotropic, woven-glass/epoxy and boron/aluminum are having very less values of above ratios, whereas remaining materials have higher ratios. All the materials shows increase in SCF values for the corresponding increase in T/A ratio. The maximum SCF value obtained for isotropic and woven-glass/epoxy is 0.42 and 0.58 at D/A=0.2 and T/A=0.10 respectively and minimum values of 0.26 and 0.25 is attained at D/A=0.7 and T/A=0.01 respectively. E-glass/epoxy, boron/epoxy, graphite/epoxy and boron/aluminum attained their maximum SCF of 0.75, 0.92, 1.02 and 0.59 at D/A=0.1 and T/A=0.10, whereas minimum SCF obtained are 0.26, 0.25 and 0.25 at D/A=0.7 and T/A=0.01 respectively.

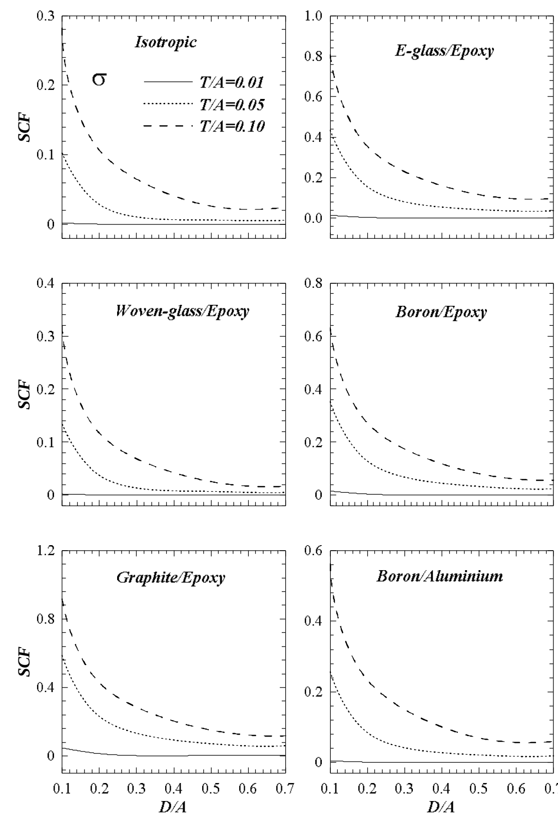

Fig. 5 Variations of SCF for (σ_z) versus D/A ratio

Figure. 5 shows the variationof SCF (σ_z)versus D/A ratio for different materials considered. The maximum SCF value obtained for isotropic, e-glass/epoxy, woven-glass/epoxy, boron/epoxy, graphite/epoxy and boron/aluminum are 0.28, 0.80, 0.31, 0.63, 0.91 and 0.56 at D/A=0.1 and T/A=0.10 respectively, whereas minimum SCF obtained is nearly zero for all materials. Thus the maximum values of SCF is obtained for graphite/epoxy and minimum for isotropic material. All the materials shows decrease in SCF values with a corresponding increase in D/A ratio and increase in SCF values with a corresponding increase in T/A ratios.

Figure. 6 shows variation ofSCF (σ_{eqv})versus D/A ratio for different materials considered. The maximum SCF value obtained for isotropic, e-glass/epoxy, woven-glass/epoxy, boron/epoxy, graphite/epoxy and boron/aluminum are 2.61, 5.47, 4.61, 7.17, 6.36 and 4.15 at D/A=0.1 and T/A=0.10 respectively, whereas minimum SCF obtained are 2.11, 2.30, 2.28, 2.41, 2.28 and 2.25 at D/A=0.7 and

T/A=0.01 respectively. Thus the maximum values of SCF is obtained for boron/epoxy and minimum for isotropic material. All the materials follows the similar trend of decrease in SCF values for subsequent increases in D/A ratios.

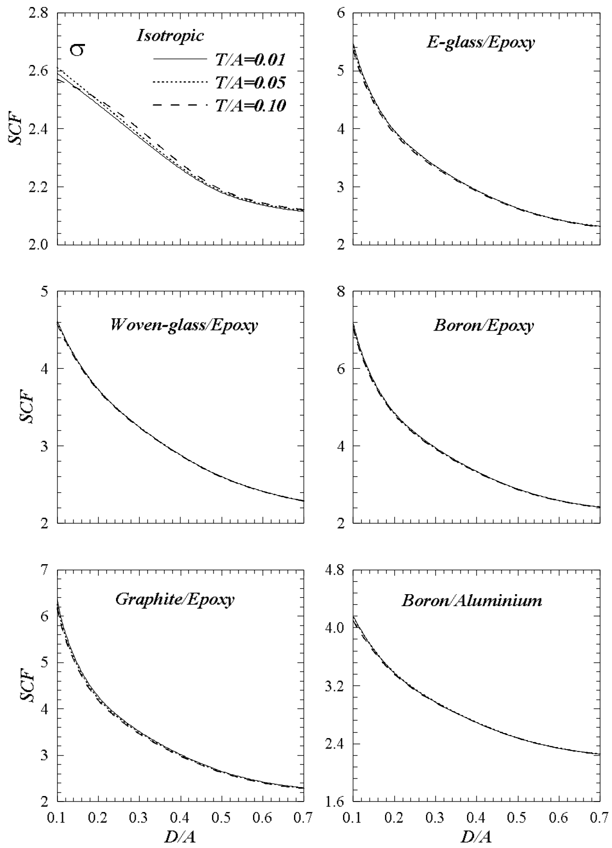

Fig. 6 Variations of SCF for(σ_{eqv}) versus D/A ratio

3. Conclusion

Following conclusions can be made from the above analysis:

i. SCF for different stresses shows the dependency of elastic constants over the varying D/A and T/A ratio.

ii. SCF for stress along x direction shows substantial variation with an increase in hole diameter, whereas very less variation is obtained for a corresponding increase in thickness of the plate.

iii. SCF for stress along y direction shows the very less effect on varying both D/A and T/A ratios.

iv. SCF for stress along the z direction shows decrease of SCF values for an increase in corresponding hole diameter, whereas on subsequent increase in thickness of the plate the SCF value increases. This proposes the need of three dimensional analysis of in-plane loading for a plate with holes.

v. SCF for von-misses stress follows the similar trend as follows for stress along x direction.

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