

## NUMERICAL INVESTIGATION ON CONJUGATE HEAT TRANSFER FROM SUDDEN EXPANSION FLOW USING NANOFLUIDS

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### ABSTRACT

Laminar two-dimensional sudden expansion flow of different nanofluids is studied numerically. The governing equations are solved using unsteady stream function-vorticity method. Conjugate heat transfer from the sudden expansion flow is reported for nanofluid. The effect of volume fraction of the nanoparticles and type of nanoparticles on heat transfer is examined and found significant impact. Local Nusselt number and average Nusselt number are reported in connection with various nanoparticle, volume fraction and Reynolds number for expansion ratio 2. Heat transfer inside and around recirculation eddy differ from rest of the channel with respect to nanoparticles and volume fraction. Symmetry plane temperature shows local peak value. Nusselt number reaches peak values near the reattachment point and reaches asymptotic value in the downstream. Bottom wall eddy and volume fraction shows significant impact on average Nusselt number. When the solid wall thickness is significant, the uniform temperature distribution inside wall is not a valid assumption. Hence the conjugate heat transfer study arises. Both solid wall heat conduction equation and heat convection in fluid region must be solved simultaneously in conjugate heat transfer problems. Along the solid wall interface, the heat flux from solid wall is same as the heat flux received by fluid region.

**Keywords:** Sudden Expansion, Nano Fluid, Recirculation, Conjugate Heat Transfer and Nusselt Number

## 1. Introduction

Sudden expansion flows are common in many industrial applications such as heat exchanger, cooling of turbine blade, combustion chambers, electronics cooling and many more. The fundamental nature of the flow separation and reattachment which occur near the sudden expansion flow made it to be the topic of many research works. The flow separation and reattachment is one of the complex phenomena and it dictates the entire flow physics in the downstream. Heat transfer characteristics in flow separation is important in engineering design aspect which leads to finding the local Nusselt number distribution and downstream temperature distribution. Conjugate heat transfer occurs when heat transfer in fluid region is coupled with heat transfer in solid region. The temperature and the heat fluxes at the solid-fluid interface are considered to be equal. The flow properties affect the heat transfer in solid region. Here the governing energy equations of solid and fluid regions have to be solved simultaneously. This is referred to as the fourth-kind boundary condition [1]. Recently conjugate heat transfer

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study is getting more attention by the researchers. Many publications are devoted to conjugate heat transfer on flat plate [2–5]. Vynnycky et al. [6] presented closed form relations for interface temperature, local Nusselt number and average Nusselt number for laminar flow over a flat plate as conjugate case. Chiu et al. [7] studied conjugate heat transfer of horizontal channel both experimentally and numerically. They found that the parametric study of operational parameters related to the conjugate heat transfer revealed that the addition of conjugate heat transfer significantly affects the temperature and heat transfer rates at the surface of the heated region. Rao et al. [8] presented the results for laminar mixed convection with surface radiation from a vertical plate with a heat source as conjugate case. They solved the governing equations by stream functionvorticity formulation. Jilani et al. [9] presented finite difference solution for conjugate heat transfer study on heat generating vertical cylinder. Steady [10] and unsteady [11] conjugate heat transfer from a cylinder in laminar flow are presented by Juncu. He carried out the

numerical investigation for Reynolds number ranging from 2 to 20. He followed alternating direction implicit (ADI) method for solving the governing equations. Kanna and Das [12] presented closed form solution for laminar wall jet flow as conjugate case. They have reported closed form solution for local Nusselt number, interface temperature and average Nusselt number and validated the same by numerical simulations. Kanna and Das [13] . Conjugate heat transfer study on laminar wall jet over backward-facing step is reported by Kanna and Das [14]. They found that the conjugate interface temperature value decreases along the step length and height. After expansion from the step its value is reduced to a minimum value followed by an increase.

Local Nusselt number has peak value near the inlet due to entrainment and second peak occurs after reattachment of the jet.In plane symmetric sudden expansions, the flow shows asymmetry for Reynolds number greater than a critical value. In an early study, Durst et al. [15] experimentally demonstrated that the asymmetry of a bifurcated flow at sudden expansion can be interchanged between the symmetric walls based on instabilities developed in the separated shear layer and the inlet flow conditions. The flow characteristics and the relation between the flow geometry and the critical Reynolds number have been studied in numerous research works. The works showed that the flow primarily depends on the Reynolds number, channel aspect ratio and channel expansion ratio. Numerical issues in accurate sudden expansion flow predictions were examined by [16] and [17]. Battaglia [17] suggested to use the effective expansion ratio instead of the actual expansion ratio to model the geometry for predictions by two-dimensional numerical models. [18] and [19] addressed mixed convection aspects of sudden expansion flow.

Recently, increasing attention has been focused on usage of nanofluids for heat transfer enhancement due to their many inherent desirable characteristics such as ability of increasing heat transfer rates without considerable pressure drop penalties. The heat transfer with nanofluids has been reviewed in many reports. It is evident from the published works that the nanofluids are one of the potential solutions for many industrial heat transfer problems. In that aspect, the effect of nanofluids in the low heat transfer zones such as flow separation regions was studied by E. Abu-Nada [20]. The author studied forced convection backward facing step flow using various nanofluids. The detailed results showed that higher thermal conductivity of nanofluids reduces heat transfer in recirculation zones and enhanced in the other regions. In a similar relevant study, A. Al-aswadi [21] highlighted the effects of nanofluids on the flow over a backward facing step for various nanoparticles.

Recently Mohammed et al. [22]reported mixed convection heat transfer from step flow. They reported that nanofluid results without second recirculation wall produce higher Nusselt number. They found that low thermal conductivity material (SiO2)results high Nusselt number at the downstream of recirculation. There are many investigation are reported for nanofluid in circular pipe flow [23], [24], [25] and [26]. Santra et al. [27] numerically studied Copper based nanofluid in heated parallel plates in the laminar range. They tested the fluid as Newtonian and non-Newtonian model and found heat transfer enhancement on both cases.

In the literature, the symmetric sudden expansion flow has been studied extensively in many studies due to the important fundamental nature of the flow and heat transfer involved. The present investigation is aimed to find the conjugate heat transfer characteristics for sudden expansion flow problem using nanofluids.

# 2. Mathematical Formulation and Numerical Procedure

The geometry and boundary conditions for conjugate sudden expansion flow is shown in Fig. 1. When the solid wall thickness is significant, the uniform temperature distribution inside wall is not a valid assumption. Hence the conjugate heat transfer study arises. Both solid wall heat conduction equation and heat convection in fluid region must be solved simultaneously in conjugate heat transfer problems. Along the solid wall interface, the heat flux from solid wall is same as the heat flux received by fluid region. The geometry and boundary conditions for conjugate sudden expansion flow is shown in Figure 1.



#### Fig. 1 The Schematic Diagram of Sudden Expansion Flow Heat Transfer by considering Conjugate Heat Transfer Condition

The non-dimensional governing equations in stream function-vorticity form are Stream function Equation

$$\nabla^2 \psi = -\omega \tag{1}$$

Unsteady vorticity equation

$$\frac{\partial \omega}{\partial t} + \frac{\partial (u\omega)}{\partial x} + \frac{\partial (u\omega)}{\partial y} = \frac{1}{\operatorname{Re}(1-\phi)^{2.5} \left[ (1-\phi) + \phi \frac{\rho_s}{\rho_f} \right]} \nabla^2 \omega \tag{2}$$

Energy equation

$$\frac{\partial\theta}{\partial t} + \frac{\partial(u\theta)}{\partial x} + \frac{\partial(v\theta)}{\partial y} = \frac{1}{\frac{1}{RePr\left[(1-\phi) + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]}}\nabla^2\omega$$
(3)

The variables are scaled as  $u = \overline{u}/\overline{u_o}$ ;  $v = \overline{v}/\overline{u_o}$ ;  $x = \overline{x}/H$ ;  $y = \overline{y}/H$ ;  $t = \overline{t}/(H/\overline{u_o})$ ;

The over bar indicates dimensional variable.  $\phi$  is volume fraction of nanoparticles;  $\rho_s$  is particle density;  $\rho_f$  is fluid density; Re is the Reynolds number defined as  $Re = \bar{u}_o H/v$ . Here  $\bar{u}_o$  is the average horizontal fluid velocity at the orifice; *H* is the channel height downstream the orifice and *v* is the fluid kinematic viscosity.

Skin friction co-efficient for nano fluid is calculated in non dimensional form

$$Cf_{nf} = \frac{2}{Re} \frac{\partial u}{\partial y} \frac{1}{(1-\varphi)^{2.5}} * \frac{1}{[(1-\varphi)+\varphi\frac{\rho_s}{\rho_f}}$$
(4)

All the solid walls are maintained at constant wall temperature ( $\theta = 1.0$ ) and nanofluid enters at ambient condition ( $\theta = 0.0$ ). Useful quantities such as local Nusselt number is evaluated from

$$N_{u} = -\frac{1}{(\theta_{\omega} - \theta_{b})} \frac{k_{nf}}{k_{f}} \frac{\partial \theta}{\partial y} \omega$$
(5)  
where  $\theta_{b}$  is bulk temperature.

Average Nusselt number is evaluated from

$$\overline{Nu} = -\frac{1}{L} \int_{0}^{L} Nu(x) dx \tag{6}$$

Unsteady heat conduction equation along the solid wall,  

$$\frac{\partial \theta_s}{\partial t} = \nabla^2 \theta \frac{\rho C_p}{k}$$
(7)

The unsteady solid wall heat conduction equation becomes,  $\theta_{ii}^{n+1} - \theta_{ii}^{n}$ 

$$\left[\frac{\frac{\partial t_{j}}{\partial t}}{\frac{\partial t}{\Delta x^{2}}} + \frac{\frac{\partial t_{j+1} + \theta_{ij-1} - 2\theta_{ij}}{\Delta y^{2}}}{\frac{\partial y^{2}}{\partial y^{2}}}\right]^{n}$$
 (8)

Where, n is current level value and n+1 is next time level value

$$\frac{\theta i j_s^{n+1} = \rho C_p \left[ \frac{\theta_{i+1j} + \theta_{i-1j} - 2\theta_{ij}}{\Delta x^2} + \frac{\theta_{ij+1} + \theta_{ij-1} - 2\theta_{ij}}{\Delta y^2} \right]_{solid}^n \cdot \Delta t + \theta_{ij}$$
(9)

Unsteady heat conduction equation for fluid region is follows,

$$\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial x} + v \frac{\partial\theta}{\partial y} = \frac{1 k_{nf}/k_f}{Re Pr\left[(1-\phi) + \phi \frac{(\rho C p)_s}{(\rho C p)_f}\right]} \cdot \nabla^2 \theta$$
(10)

The boundary conditions applied in the present numerical simulation are shown in Figure 1. At the inlet orifice, parabolic horizontal velocity profile is assumed with an average velocity of  $u_{o}$ , and the fully developed flow conditions are applied at the channel exit. No slip and no penetration conditions are applied for velocity components along all the solid walls. The boundary conditions for the flow study are as follows:

At the entrance 
$$(x = 0)$$
:  
 $u(y)= 12y - 24^2$ ,  $\psi(y) = 6y^2 - 8y^3$ ,  $\omega(y) = 48y - 12$ 
(11)

At the exit (x = 30):

$$\frac{\partial^{2\varphi}}{\partial x^{2}} = 0, \frac{\partial \omega}{\partial x} = 0 \quad \frac{\partial u}{\partial x} = 0$$
 (12)

On solid boundaries u=0 and v=0

u=0 and v=0 (13) The boundary conditions for the heat transfer study are as follows:

At the entrance (x = 0):  $u(y)= 12y - 24^2$ ,  $\psi(y) = 6y^2 - 8y^3$ ,  $\omega(y) = 48y - 12$ (14)

At the exit 
$$(x = 30)$$
:  
 $\frac{\partial^2 \psi}{\partial x^2} = 0, \frac{\partial \omega}{\partial x} = 0, \frac{\partial u}{\partial x} = 0 \frac{\partial \theta}{\partial x} = 0,$ 
(15)

Along solid boundaries

 $u = 0, v = 0, \theta = 1.0$  (16) At interface region along solid wall and fluid region interaction the following boundary condition is considered for the present simulation,

$$\left(\frac{\partial\theta}{\partial y}\right)_{solid} = \left(\frac{K_{nf}}{k_s}\frac{\partial\theta}{\partial y}\right)_{fluid}$$
(17)

Eqn (18) becomes  

$$\frac{k_f}{k_s} = \frac{k_{nf}}{k_s}$$
(19)

$$\left( K_s \ \frac{\partial \theta}{\partial y} \right)_{\substack{y=0\\ Solid}} = \left( K_{nf} \ \frac{\partial \theta}{\partial y} \right)_{\substack{y=0\\ fluid}}$$
(20)

The governing equations are discretised on rectangular grids. The central differencing scheme is adopted for both the convective as well as the diffusive terms (Roache (1998)). Stream function equation, Eqn. (1), is solved by Gauss-Seidel method. The unsteady vorticity transport equation, Eqn. (2), is solved by Alternate Direction Implicit scheme (ADI) (Roache

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(1998)), along with Eqn (12), until the steady state solutions are arrived. At every time step the residue of the discretised equations are monitored and the solution is accepted only when the residues become negligibly small (less than 0.001). The numerical values of u and vvelocities at every time step are monitored to see the changes with time marching. When there are no changes in the velocity variables and vorticity with further time steps, the solution is considered as the steady state solution. In addition, total kinetic energy which is calculated as the sum of  $(u^2 + v^2)^{1/2}$  at all the discrete nodes is monitored with time marching. At the steady state, this quantity is also confirmed to shows no change with further time steps. This reflects that the domain reaches steady state total kinetic energy value. For conjugate heat transfer process, the energy equations in the fluid region and in the solid region are solved simultaneously. Since nano fluid is not reported in the open literature for sudden expansion flow the code is validated for nano fluid by using two similar benchmark simulations of backward step flow which encounters flow separation, recirculation and wall bounded flow which are similar to sudden expansion flow.

## 3. Code Validation and Grid Independence Study

Sudden expansion flow without nano particles are simulated for Re=70 and 610 and compared with the results of Durst *et al* [15] for expansion ratio of 2. The results were found good agreement for both numerical and experimental results [28].



Fig. 2a Cu nanoparticles Re= 400,  $\phi$ =0.2: X<sub>r</sub>= 5.2 [17] and present X<sub>r</sub>= 5.137: Streamline contour

As the first problem, the results of [17] for backward facing step (BFS) flow with Copper nanoparticles of volume fraction of 0.2 for Re = 400 is selected for comparison. The recirculation length predicted by present code (x = 5.137) matched well with that of [17] (x = 5.137) Fig. 2a. The present result is within 1% of the reference study.

The code is further checked for simulations of flow with SiO<sub>2</sub>nanoparticles for Re=100 and volume fraction of 0.05, and compared to the results of Al-aswadi et al. [21] Fig.2b and Fig.2c. To validate the energy solution procedure forced convection from backward step flow using Copper nano particle is solved for Re=200 and  $\phi = 0.2$  and compared with Abu Nada [20] Fig.2d and Fig.2e. It can be seen from the Fig. 2a that the present code results are within acceptable deviation limit.



Fig.2b Sio<sub>2</sub> Nanoparticles, Re=100,  $\phi$ =0.05 and X/s = 1.04



Fig. 2c Sio<sub>2</sub> Nanoparticles, Re=100,  $\phi$ =0.05 and X/s =32.8



Fig. 2d Cu nanoparticles, Re=200,  $\phi$ =0.2: Bottom wall Nusselt Number



Fig. 2e Cu Nanoparticles, Re=200,  $\phi$ =0.2: Top wall Nusselt number

The detailed validation of hydrodynamic results for BFS problem as well as sudden expansion flow using nano fluid can be found from David et al [29]. Different grid systems are tested to find suitable grids to simulate the present problem. Grid independence test was carried based on bottom wall reattachment length for Cu nano particles for Re= 200 and  $\phi = 0.2$  and average Nussult number. Grids 201 × 61 which is less than 1% variation in average Nusselt number with 251 × 121, is chosen for the simulation of entire study.

## 4. Results and Discussion

Conjugate heat transfer study from sudden expansion flow is reported for nano fluid. First hydrodynamic solution is obtained by ADI method. Cu based nano fluid is considered for the entire study. Further Brass based solid slab of thickness of 0.5H is provided at the bottom and top of the channel to investigate the conjugate effect. Results presented for isotherm contour from top, bottom solid wall and channel flow of the fluid. The conjugate interface temperature along bottom and top wall is also reported to understand the significance of wall thickness.

**Table1. Properties of Various Materials** 

Properties	Cu	$Al_2O_3$	SiO <sub>2</sub>	$H_2O$	Brass	Glass wool
K	400	40	1.2	0.613	109	0.04
Ср	385	765	703	4179	377	
ρ	8933	3970	2200	997.1	8400	

Fig. 3 shows the temperature contour for conjugate heat transfer from sudden expansion without nano particles. It depicts the isotherm from top and bottom solid wall section and channel also. Figure 3a shows the top wall temperature contour. The top surface of this section is maintained at constant temperature and bottom is representing conjugate interface with channel. It is observed that the temperature contours are less variant in *x* direction. The influence of top wall vortex is also negligible. Due to bottom wall vortex, the isotherm are reflected near inlet and further in downstream it turns into parallel to each other as shown in the channel region (Fig 3b). Fig. 3c shows the temperature contour from bottom solid wall region.

It is noticed that near inlet region the temperature contour shows the two dimensionality. Bottom wall vortex attributes this behavior. However in the downstream the temperature contour is parallel to each other.



Fig. 3a Top Solid Wall Region Temperature Contour ( $\phi$ =0)



Fig. 3b Channel Region Temperature Contour



Fig. 3c Bottom Solid Wall Region Temperature Contour



## Fig. 3 Isotherm Contour of Sudden Expansion with Zero Nano Particles

Fig. 4 show the effect of nanoparticle in the conjugate heat transfer. Isotherm presented for the volume fraction equals 0.1. The presence of nanoparticles increases the magnitude of heat transfer in the fluid region. This hike in temperature increases the temperature in both top and bottom wall region. The two dimensionality is found in the bottom wall region (Fig.4.c). Further increase in volume fraction leads to higher temperature values.



Fig. 4a Top solid wall region temperature contour





Fig. 4b Channel Region Temperature Contour



Fig. 4 Isotherm Contour of Sudden Expansion with Cu Nano Particle Re =200 and  $\phi = 0.1$ 



Fig. 5a Top Solid Wall Region Temperature Contour



Fig. 5b Channel Region Temperature Contour



Fig. 5. Isotherm contour of sudden expansion with Cu nano particle Re=200 and  $\emptyset = 0.2$ 

Fig.5. shows the effect of volume fraction at 0.2. It is noticed that higher volume fraction causes larger eddy size which in turn increases the heat transfer inside the channel.

### 4.1 Effect of Reynolds Number

Effect of Reynolds number in conjugate heat transfer are shown in Figures 6-8. For Re=70 it is found that symmetry in the temperature contour inside the channel. In solid region the temperature contours shows one dimensional behavior. When Reynolds number increases to 100 symmetry in temperature contour is observed near inlet in the channel region. In solid wall near inlet the isotherm shows two dimensionality in the bottom wall. It is noticed that temperature variation occurred mostly in the normal direction of solid wall. Higher Reynolds number (Fig. 8) depicts variation in the temperature contour. Particularly the temperature is increased in the recirculation region more than the rest of the place.



Fig. 6a Top Solid Wall Region Temperature Contour



Fig. 6b Channel Region Temperature Contour



Fig. 6 Effect of Reynolds Number on Conjugate Heat Transfer Study Cu and  $\emptyset = 0.2$ : Re = 70



Fig .7a Top solid Wall Region Temperature Contour



Fig. 7b Channel Region Temperature



Fig. 7 Effect of Reynolds Number on Conjugate Heat Transfer Study Cu and  $\emptyset = 0.2$ : Re = 100







Fig. 8b Channel Region Temperature Contour



Fig. 8 Effect of Reynolds Number on Conjugate Heat Transfer Study Cu and  $\emptyset = 0.2$ : Re = 200

### 4.2 Conjugate Interface Temperature

Top and bottom wall interface temperature is plotted for better understanding the temperature distribution while solid wall thickness is non zero. Figure 9 shows the conjugate interface temperature for Newtonian fluid at Re=200. Along the bottom wall temperature is decreased to a local minimum value up to reattachment point and further increases to a maximum value and further it turns to an asymptotic value. Whereas the top wall interface temperature is decreased to a minimum value up to top wall vortex reattachment point and further increases to a maximum value and in downstream it reaches an asymptotic value. Also it is noticed the within the recirculation region the bottom wall interface temperature is higher than the top wall temperature. But beyond reattachment point the trend is reverse. When Cu nano fluid is introduced the local minimum temperature value is increased for top wall interface temperature as well as bottom wall interface temperature (Fig 10) for volume fraction equals 0.1. When volume fraction is further increased to 0.2 near recirculation region both wall interface temperature increases to a local peak and a kink is appeared near reattachment location. But in downstream they spread parallel to each other.



Fig. 9 Conjugate Interface Temperature: Zero Nano Particle Re=200





Fig. 10 Conjugate Interface Temperature: Cu Nano Particle, Re = 200 and  $\phi = 0.1$ 



Fig. 11 Conjugate Interface Temperature: Cu Nano Particle, Re = 200 and  $\phi = 0.2$ 



Fig. 12 Conjugate Interface Temperature: Cu Nano Particle, Re = 70 and  $\emptyset = 0.2$ 



Fig. 13 Conjugate interface temperature: Cu nano particle Re = 100 and  $\phi = 0.2$ 



Fig.14 Conjugate interface temperature: Cu nano particle, Re = 200 and  $\phi = 0.2$ 

Figures 12-13 show the effect of Reynolds number on conjugate heat transfer. When Re=70 near inlet both wall interface temperature approaches same value. Further in the downstream direction of the channel they spread parallel to each other. The bottom wall vortex reduces the bottom wall temperature. For higher Reynolds number (Fig 14) the trend is similar but the value of local interface temperature is increasing.

### 4.3 Local nusselt number

The Nusselt number distribution from the bottom wall is presented for effect of nano particle and effect of Reynolds number. Fig 15 compares the effect of Cu nano particle on local Nusselt number for Re = 200 and volume fraction is 0.2 case. Local Nu is increased for zero nano particle fluid to a local peak value and further decreased in the downstream location. Beyond reattachment location it reaches an asymptotic value. When nano particle is introduced the trend is same as the earlier but the local values are increased. The effect of Reynolds number is shown in Fig. 16. It is observed that when Reynolds number equals to 200 there is a kink occurred after reattachment point. Fluid separates at the corner of the channel and reattach in the bottom wall. After reattachment it retards at high Reynolds number. This attributes in heat transfer which results a kink in local Nusselt number.



Fig. 15 Local Nusselt Number from Bottom Wall: Effect of Nano Particle



Fig. 16 Local Nusselt Number from Bottom Effect of Reynolds number

### 5. Conclusion

Two dimensional conjugate heat transfers are numerically investigated for sudden expansion flow using nano fluid. At low volume fraction it is found that the temperature variation occurs in normal direction rather stream wise direction in the solid wall. For higher volume fraction the interface temperature is increasing. Due to the recirculation vortex the bottom wall interface temperature is less than top wall interface temperature. At low Reynolds number symmetry in the isotherm is noticed. Increment in Reynolds number increases interface temperature. Local Nusselt number is reported for effect of volume fraction and Reynolds number. Nu increases to a peak value and decrease to a local minimum value. Further in the downstream channel direction it reaches an asymptotic value

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