MECHANICAL VIBRATION ANALOGY OF A MANUFACTURING SYSTEM

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ABSTRACT

Complexities of manufacturing system have been modeled by various modeling techniques viz. system dynamics modeling, discrete event simulation, mathematical models and various analogies. Manufacturing system continues to produce, in response to customer orders or stocking needs. This behavior is similar to mechanical vibration system which continues to vibrate in response to varying external force. Present work is an attempt to characterize manufacturing system in terms of mechanical system. Variation of displacement with time in mechanical system and variation of manufacturing in industry exhibits strong similarity. These plots must have similar root causes for matching behavior. This similarity between two plots forces modeling of one system into another. This concept is to visualize industrial system by way of equivalent mechanical system. The modeling is achieved by searching (m, c, k) values of equivalent system. 3D search method is used to optimize the system properties. The model once developed can be manifested for enforcement of specific conditions of inputs or clarification of system properties for isolated effects.

Keywords: *Manufacturing System, Mechanical Vibration and Dynamic Modelin*g

1. Introduction

Industrial manufacturing is not steady flow. The output varies from time to time. Daily, monthly, quarterly or seasonal variations are observed in output. Factors responsible for such variation are the orders received; inventory level, breakdown maintenance, human resource inputs and opportunities. Measurement of such inputs in quantitative terms is difficult task. Industrial output is tangible. Its dynamic behavior maps to curvilinear graph with many uneven ups and downs.

In addition to mathematical modeling, system dynamics, discrete event simulation there had been attempts to understand manufacturing system by establishing its analogy with another system. Funnel analogy has been used to explain a method that determines throughput time on a shop floor by controlling the amount of jobs released and thus, the input to the manufacturing system [1, 8]. Analogies between manufacturing and concepts in physics, such as turbulent flow [7] and transport phenomenon for Production networks and supply chains [2] have beenalso presented in the past. Sirazetdinov T.A. [3]proposed spring and inertia system analogy of industrial object but the paper does not mention about damping in manufacturing system. Chryssolouris et al. [4] used the mechanical analogy concept to assess the flexibility of a manufacturing system wherein behavior of manufacturing system is described with the attributes of a mechanical system viz. its inertia, damping and stiffness. Their approach computes the damping factor of a manufacturing system, from the eigen-values of the transfer function in the frequency domain. Motivation behind establishing analogy between manufacturing and mechanical system is the difficulty in describing flexibility in quantitative terms in context of manufacturing system and same being well defined in case of mechanical system [5]. Eigen value approach for system identification is also explained by Maia and Silva [11]. A mechanical system has force varying over time as an input and an output is the displacement that depends upon the characteristics of the system's elements. In similar way manufacturing system receives supply orders as input and production varying against time as output. Production is assumed to rise up and decline at certain rates based on order quantity and due supply period. Mapping this on mechanical system response help to yield mass, stiffness and damping coefficients of manufacturing system [6].

2. Mechanical Vibration System

Equation of motion for single degree of freedom mechanical vibration is

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$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (1)$$

Where x is displacement, $\dot{x}$ is velocity, $\ddot{x}$ is acceleration, m is mass, c is damping coefficient, k is stiffness of spring and F (t) is input excitation in context to mechanical system. The equation of motion (1) can be solved by using ordinary differential equation. Fourier or Laplace transform can also be used to get the solution.

In analysis of response, impulse excitation can be considered as basic excitation. Other excitations like step, ramp, harmonic or random can be obtained by integration of impulse excitation. Unit impulse excitation function known as Dirac-delta function is defined mathematically as

$$\begin{cases} \delta(t-a) = 0 \text{ for } t \neq a \\ \int_{-\infty}^{\infty} \delta(t-a)dt = 1 \end{cases} \quad (2)$$

Natural frequency of vibration (ω_n) and damping factor (ξ) are derived from physical properties namely mass, stiffness and damping coefficients. The derived properties are more useful for computation. Moreover they are helpful in visualization of physical behavior.

$$\omega_n = \sqrt{\frac{k}{m}} \quad (3)$$

$$\xi = {}^{c}/_{2m\omega_n} \quad (4)$$

The term ξ and ω_n are combined by following expression to give frequency of damped oscillation ($\omega_{d.}$)

$$\omega_d = \omega_n\sqrt{1-\xi^2} \quad where\ \xi < 1 \quad (5)$$

Value of ξ classifies the vibration as undamped if $\xi = 0$, as underdamped if $\xi<1$, as critically damped if $\xi=1$ and overdamped if $\xi>1$. The impulse response (displacement) in underdamped vibration is given by

$$g(t) = \begin{cases} \frac{1}{m\omega_d} e^{-\xi\omega_n t} sin(\omega_d t) & for\ t > 0 \\ 0 & for\ t < 0 \end{cases} \quad (6)$$

Any vibration system in general can be considered to be subjected to random excitation. As shown in Figure 1, random excitation can be assumed to be built by series of impulse excitations applied on the mechanical system for very small time Δt. Response at time 't' to random excitation is determined by integrating response at time '(t-τ)' for impulse excitation applied at time 'τ'

$$x(t) = \int_0^t F(\tau).g(t-\tau)\,d\tau \quad (7)$$

This is also called Duhamel's integral or convolution integral.

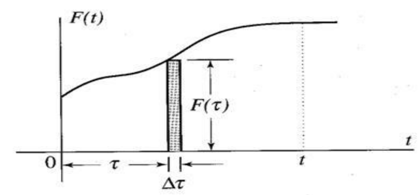

Fig. 1 Random Excitation as Series of Impulse Excitations

3. Analogy of Manufacturing and Mechanical System

Variation of displacement with time in mechanical system and variation of production in industry exhibits strong similarity. These plots must have similar root causes for matching behavior. This similarity between two plots forces modeling of one system into another. This concept is to visualize industrial system by way of equivalent mechanical system. The modeling is achieved by searching (m, c, k) values of equivalent system.

In mechanical system, mass or inertia resists movement. It moves reluctantly when acted upon by unbalanced force. Spring is used to store and release energy. Significance of vibration analysis is because of existence of damping. Role of damper is to oppose the direction of motion. Damping in mechanical system is friction. Portion of input excitation is lost in overcoming friction or damping.

Displacement is treated as output or response variable. It changes with time under the influence of activation force F (t).

In manufacturing system number of units produced in unit time is its displacement. Supply orders from market or storage for inventory is the excitation force acting on the industrial object. Land, plant, machinery and other assets of the industrial object by and large constitutes the mass. The resources namely men, material, money etc. exhibit spring like behavior as while getting consumed for production, these resources re-energize for further production. Activation force attracts the system response along with undesirable effect called damping.

Damping of industrial system comprises of inefficient planning, breakdowns, absenteeism and lack of material etc.

4. Discrete Method for Computing Response in Mechanical System

The displacement at time t is obtained using equation (6). The input function is assumed to be the impulse of fixed duration. Any random excitation can be split into series of impulses. Procedure for computation of response involves complicated integrations. It can be simplified for impulse inputs by equivalent summation. Implicitly defined excitation does not provide direct definition in terms of time. This increases the degree of difficulty while integrating such function. The most viable option is to obtain response using numerical evaluation of convolution integral. To compute the response on computer it is necessary to discretize the time (Figure 2). Excitation forces F (n) at those discrete instances of time are used to obtain discrete response. The discrete-time impulse response is

$$g(n) = \frac{1}{m\,\omega_d} e^{-n\xi\omega_n T} sin(n\omega_d T) \qquad (8)$$

Where, n $\in$(1, 2, 3 …k)

'T' is sampling time. Convolving F (n) with g (n) for n $\in$(1, 2, 3……………k) generates the response curve.

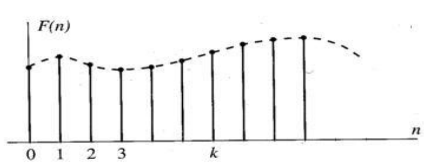

Fig. 2 Discretizing Excitation

To find the mechanical system equivalence of manufacturing system is a turnaround problem. From the response and input excitation, system properties viz. mass, stiffness and damping coefficient are to be determined. Ratio of order quantity and due supply period excites the system to continue producing product. The quantity produced per day is the response (displacement) of the system.

5. Algorithm for Determination of (m, c, k) Values

Mass, damping factor (zeta) and natural frequency (omegan) of mechanical system equivalent to manufacturing system are found by mapping response curve generated for values of 'mass', 'zeta' and 'omegan' on actual response curve. Optimization algorithm is used to search values of 'mass', 'zeta' and 'omegan'.

The objective function used to determine 'mass', 'zeta' and 'omegan' is minimization of root mean square error (rmserror) between computed and actual response. To find 'rmserror', response for set of values of 'mass', 'zeta'and 'omegan' is to be computed. Damped frequency of oscillation 'omegad' is calculated using equation

Omegad = omegan*(1-zeta^2)^0.5

Set of excitation forces are retrieved in sequence and stored in array named force. The array of transfer function g is obtained using equation

*g(i)=(1/(m*omegad))*Exp(-(i)*zeta*omegan*delta)*
Sin(i * omegad)*

Displacement array is then obtained by convolving force [] array with g [] array. This displacement array is the computed response (Refer Figure 3). Square-root of sum of square of difference between computed response and corresponding actual displacement divided by number of records is 'rmserror'. Convolution operation calculates displacement at any time 't' as a sum of product of each 'force(i)' for 'i' less than or equal to 't' and transfer function 'g(j)' such that i + j= t. By using convolution, displacement array is easily obtained by algorithm shown in Figure 4.

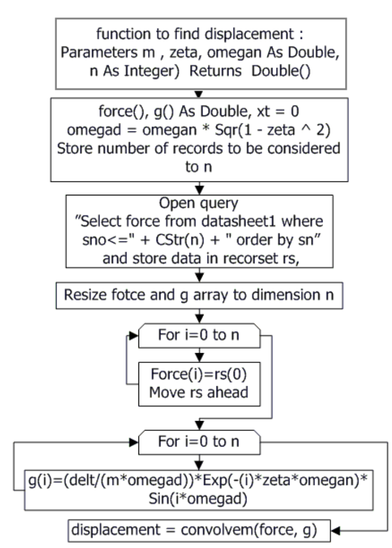

Fig. 3 Function to find Displacement

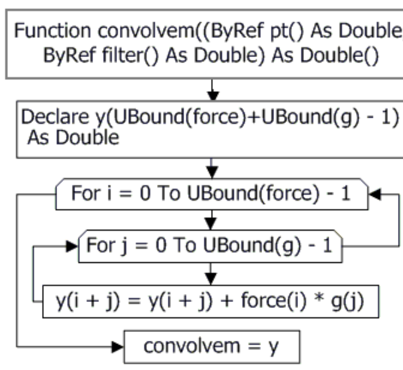

Fig. 4 Function for Convolving Two Arrays

6. Conclusion

In present paper, analogy between the dynamic behavior of manufacturing system and the single degree of freedom mechanical vibration system is drawn and approach for estimating mass, damping factor and natural frequency for the manufacturing system is discussed. Representation of manufacturing system by simple spring-mass-damper system will help in using knowledge from mechanical vibration system for analysis of complex production system. Changes in system properties in every quarter may lead to some useful information about working of manufacturing system. Damping factor in particular may help in indicating gross inefficiency in manufacturing system.

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Nomenclature

Symbol	Meaning	Unit
k	spring stiffness	N/mm
m	mass	kg
ω_n	natural frequency	Hz
ω_d	damped frequency	Hz
c	damping coefficient	-
T	sampling time	min
ξ	damping factor	-

