



USE OF ANOVA FOR TESTING REPRODUCIBILITY - A CASE STUDY

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ABSTRACT

ANOVA is the method to test a hypothesis. It is statistically based objective decision making tool for detecting any differences in average performance of the groups of items tested. So far, researchers used ANOVA to find out the existence and extent of relation between the chosen input parameters and selected output parameters. On the contrary, In this work the authors tried to apply ANOVA to test the reproducibility of results when the relation between the input and out parameters is already known. One of the cases that needs checking of reproducibly of results is the validation of the numerical results with experimental ones. In this work the force obtained from simulation is to be compared with the experimental results for which the experimental results are to be reproducible for the validation to be valid. ANOVA is used here to check whether the variation among the averages of the groups is more than individual variation taking BHF (blank holding force) and punch force as input and output parameters respectively.

Key words: ANOVA, Reproducibility and Validation

1. Introduction

Analysis of Variance (ANOVA) a method developed by Ronald Fisher, compares the individual variation with the variation of averages and is used to prove hypotheses. It is a decision making tool that takes the variation into account rather than using the pure judgment [1]. Many researchers [2-11] used this method to establish the relation between the chosen input parameters with the output parameters and to find out the quality characteristic that influences the output characteristic the most. But in this work ANOVA is tried for a different purpose i.e to validate the numerical model with the experimental one.

Validation is considered to be a confirmation process that a model can adequately predict the underlying physics. A valid model must well approximate the physical behavior with a satisfactory level of accuracy [12]. In various disciplines, a number of approaches have been utilized to validate a model. Out of various approaches, a simple comparison between simulated and experimental results is the most straightforward approach [13-17]. Similar to the numerical model involving errors like discretisation, truncation etc., there exists experimental errors such as uncertainty in the work piece properties, varying lubricating conditions, hysteresis of the measuring equipment etc. So as to validate the numerical model with the experiments the reproducibility of the results

should be ensured. In this work the reproducibility of the results is checked using ANOVA.

2. Experimental Setup

A hydraulic press of 25 ton capacity, Shown in Fig 1, is used for conducting the experiment. The schematic diagram of the experimental set up is shown in Fig 2.

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Fig. 1 Experimental Setup

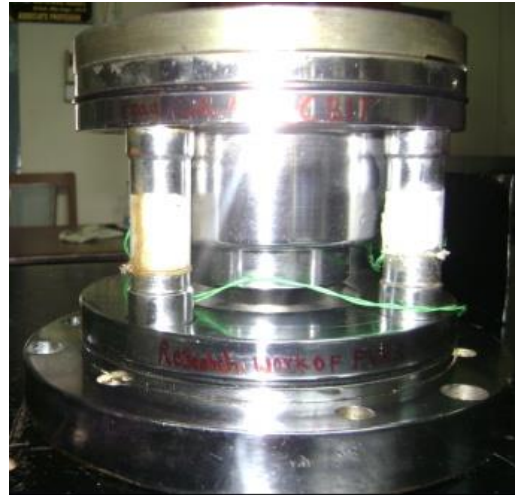


Fig. 3 Load Cell

The die is fixed in the load cell, which is interfaced to computer through digital force indicator, signal conditioner, data acquisition system and I/O card. Software compatible with the interfacing unit is loaded in computer for obtaining transient force response curves of the process.

The load cell shown in figure 3 is specially designed to incorporate dies on a die holding plate. It consists of four columns below the die holding plate. Strain gauges are bonded to the columns.

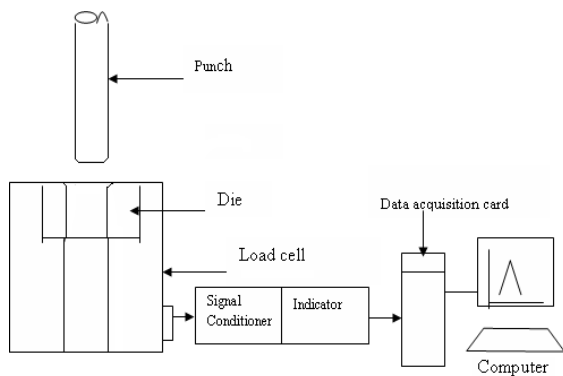


Fig. 2 Schematic Diagram of Experimental Setup

The columns get elastically compressed thereby changing the resistance in the strain gauges. The load cell gives proportionate electrical signals to signal conditioner, where it is amplified in acceptable form and is sent to the digital force indicator which is connected to the computer through data acquisition card.

The software is written in Visual Basic and is menu driven. The software provides provision for either viewing the data or viewing the plot of load v/s time and highlights the peak force applied during the Metal Forming Operations.

3. Experimental Procedure

The dependent, independent, control and extraneous variables are listed in table 1.

Table 1: Independent, Dependent, Controlling and Extraneous Variables in Experimentation

S.No	Type of variable	Variable	Value
1.	Independent Variable	Blank holding force	-----
2.	Dependent Variable	Drawing force	-----
3.	Controlling Variables	Punch diameter	25 mm
		Blank diameter	45 mm
		Blank thickness	1 mm
		Punch profile	2 mm
		Die Profile	3 mm
		Clearance	10%
		Lubricant	Deep drawing oil

4.	Extraneous variables	Material Variables Extent of lubrication	----- of -----
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Initially the press is operated with out keeping the blank thereby obtaining the blank holding schema. The purpose of obtaining blank holding schema is to use it as input for simulation. Three such schemas are considered for the validation. Blank holding schema is varied by changing the initial spring force by adjusting the lock nut provided on the flange placed on the top of the spring.

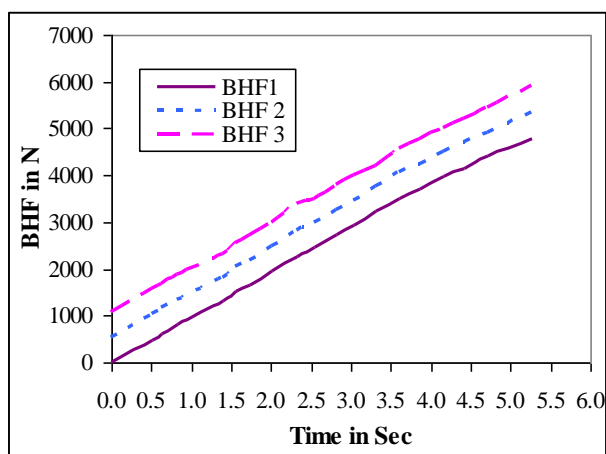


Fig. 4 BHF Schema Obtained from the Experiment

The blank holding force obtained with the time of stroke is shown in Fig.4. Since it is spring loaded blank holder, the schema should be linear. The same is evident from the graph with little variation due to experimental error.

Blanks of 45mm diameter are cut on the mechanical press using simple blanking die. For lubrication deep drawing oil is applied as lubricant on the both sides of the blank and placed it on the drawing die. Five cups are drawn at each blank holding force and load v/s time of stroke diagram is plotted.

4. Results and Discussion

A sample of blank and cup drawn is shown in Fig.5. The maximum force is found out for each one and presented in table 2.



Fig. 5 A Sample of Blank and Cup Drawn

Table 2: Maximum Load at Various Blank Holding Forces

BHF	Maximum load for the cups drawn (in N)				
	1	2	3	4	5
BHF 1	11575	11492	11566	11582	11502
BHF 2	11676	11643	11642	11713	11593
BHF 3	11769	11742	11782	11773	11674

In ANOVA (Analysis of Variance) one of the mathematical assumptions is that the variance of populations that groups are sampled from are equal. Scariano and Davenport [18] demonstrated the consequences of violating the assumption. This homogeneity of variance can be estimated by sphericity measure which is more general condition of compound symmetry[19]. Compound symmetry is assumed to be met if the variances of repeated measures are approximately equal and also the covariances. The variances and covariances of the data in table 2 is computed and presented in table 3 in matrix form in which the diagonal terms are the variances and the off diagonal terms are covariances.

Table 3: Compound Symmetry

	BHF 1	BHF 2	BHF 3
BHF 1	1838.8	930	887.8
BHF 2	930	1987.3	866.3
BHF 3	887.8	866.3	1933.5

From the table 3 it is seen that the variances are ranging from 1838.8 to 1933.5 and are approximately equal and the covariances are ranging from 866.3 to 930 and are approximately equal. Hence it is assumed that the compound symmetry is met. Compound symmetry is a sufficient but not necessary condition for sphericity [20] i.e. If compound symmetry is met sphericity is ensured and sphericity is checked if and only if the condition of compound symmetry is not met. As sphericity which is the pre-condition for ANOVA is

met, ANOVA for the data in table 2 is carried out and presented in table 4.

Table 4: ANOVA Table

Source of Variation	SS	df	MS	F
Between Groups	124800.9	2	62400.47	47.11
Within Groups	15894.4	12	1324.533	
Total	140695.3	14		

In table 4, ‘SS’ is sum of squares of the factor, ‘df’ is degrees of freedom which is defined as the number of independent comparisons between the values (generally N-1). ‘MS’ is the mean of squares i.e the ratio of ‘SS’ to ‘df’ and ‘F’ statistic is the ratio of mean squares ‘between the groups’ to that of ‘with in the groups’. The calculated value of F is compared with the standard value obtained from the F-distribution curve. The F distribution is an asymmetric distribution that has a minimum value of 0, but no maximum value. The curve reaches a peak not far to the right of 0, and then gradually approaches the horizontal axis at larger F values. The F distribution approaches to zero, but never quite touches the horizontal axis on the right side. The F distribution has two degrees of freedom, *d1* for the numerator, *d2* for the denominator (in this case *d1* is the degrees of freedom of ‘between groups’ and *d2* is degrees of freedom of ‘with in the groups’).

F- Distribution is normally presented in the form of tables for different risks (0.01, 0.05 and 0.1). From the standard tables $F_{0.01,2,12} = 6.92$ [1]. Since $F_{data} > F_{table}$, the variation in the averages is more than the individual averages at 99% confidence level. The influence of BHF on the punch force is well known. So, here, F test is done not to establish the effect of BHF on punch force but to check the reproducibility of the experimental results with the given parameters. As there is considerable variation of averages of punch forces with the variation of BHF than individual averages, the reproducibility of results is ensured and the comparison of the experimental values at various BHFs can be carried out with the simulated ones.

Simulation is done using LSDYNA taking a quarter model due to axisymmetric nature of the drawing operation, with the conditions same as existing in experimentation. As stated earlier validation of the finite element model is carried out by comparing the force obtained with the experiment with that of in simulation. The finite element model is shown Fig. 6

As a sample, the experimental curve for the first sample with BHF 1 is presented in Fig. 7 and corresponding curve of simulation is presented in Fig. 8.

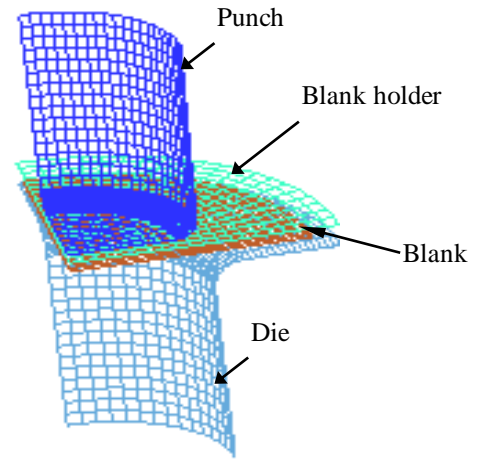


Fig. 6: Finite Element Model

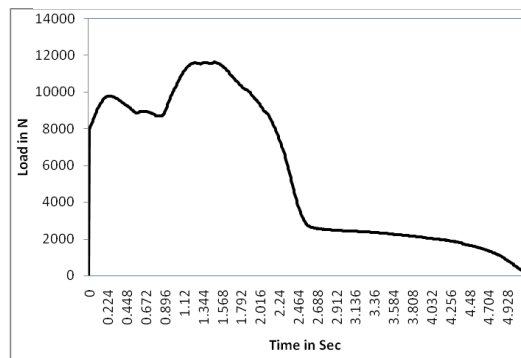


Fig. 7 Transient Load v/s Time Diagram obtained from the Experiment

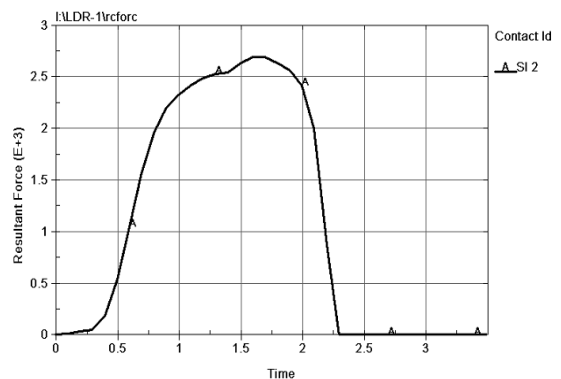


Fig. 8 Transient Load v/s Time Diagram Obtained in Simulation

For the sample presented above (Fig. 7 & Fig. 8), the maximum force obtained from the experiment is 11,575 N and from the simulation, it is found to be 2697.9 N. Since quarter model is taken for the finite element model, the actual force obtained from the simulation is four times of the given value i.e. 10,791.6 N.

Table 5: Comparison of Experimental Values with the Simulated Ones.

BHF	Avg. max. load in N	Max. load in N	Deviation in (N)	Deviation in Percentage
	(Exp.)	(Simu.)		
1	11543.4	10791.6	751.8	6.5
2	11653.4	10840	813.4	7
3	11748	10981.6	766.4	6.5

The average values obtained with the experiment and with the simulation using various blank holding forces are compared and the deviation is presented in table 5.

From the above table, it is observed that the deviation between experimental and simulated values ranges from 6.5% to 7%.

5. Conclusions

ANOVA is so far being used to test the hypotheses in which whether the assumption that a selected input parameter affecting an output parameter is true. It is also used to find out the relative contributions of various input parameters. But in this work authors used the technique to check the reproducibility of experimental results during the validation of a finite element model. It is found that the technique worked well and is very much useful in such problems. The authors recommend this statistical tool to the people working on numerical simulations, when the validation is to be carried out by direct comparison with experimental results, rather taking simply average of a few readings from repeated experiments.

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