



OPTIMIZATION OF MILLING OPERATION USING COMBINED GREY TAGUCHI RELATIONAL METHOD AND ANOVA

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ABSTRACT

In the present study, optimization of process parameters on milling machine is carried out using a combination of Taguchi method and grey relational analysis method. Speed, feed and depth of cut are taken as input process parameters and cutting force, torque and power are selected as the target values. All the three input parameters are taken at four different levels and sixteen experimental runs are performed based on L16 orthogonal array of Taguchi method. An optimum parameter combination is obtained using Grey relational analysis method. By analyzing the grey relational grade matrix, the degree of influence for each controllable process parameter onto individual target value can be found. Based on the relatively new combined Taguchi and Grey relational approach, it is found that feed has maximum influence on the target characteristics. Analysis of Variance (ANOVA) is also applied to identify the most significant factor.

Keywords: *Milling, Taguchi Method, ANOVA, Grey Relational Analysis, Normalization, Grey Relational Grade, Grey Relational Co-efficient.*

1. Introduction

In recent times, the chief aim of any industry is to manufacture low cost, high quality products in shortest possible time. As far as versatility of any process is concerned, milling is perhaps the most versatile machining operation and most of the shapes can be generated by this process [1]. Force and torque measurement and prediction has always remained a prime concern for any industry. Based on these, power can be predicted and actual cost incurred can be found. The researchers the world over have extensively worked on force, torque and power prediction and their optimization for Milling process. Yang et al [2] applied design of experiment approach to optimize surface roughness and other parameters in end milling process. It has been noted that most of the researchers have selected speed, feed and depth of cut as machining parameters [3, 4, 5, 6].

Recently, a variety of industries have employed the designs of experiments (DOE) method over the years to improve products or manufacturing processes [7]. It is a powerful and effective method to solve challenging quality problems. In practice, the DOE method has been used quite successfully in several industrial applications as in optimizing manufacturing processes or designing electrical/mechanical components. The DOE method has been successfully applied to optimize the machining parameters for electrical discharge machining of boron

carbide [8]. DOE method along with ANOVA analysis has also been applied to determine optimal settings of grinding conditions and grinding cycle time for which results were compared and analyzed [9]. DOE tools have been applied by the authors to optimize various process parameters under different working conditions on electro discharge machining process [10, 11]. Optimization of surface roughness in end milling operation has been successfully conducted using Taguchi's Design of Experiment technique [12].

Few years before, Deng [13] has proposed a Grey relational analysis which is a method for measuring the degree of approximation among the sequences using a Grey relational grade. Theory of the Grey relational analysis is found suitable for optimization of process parameters and as a result has attracted considerable interest among researchers. The method has been applied for optimization on die sinking EDM parameters [14], injection molding process [15] and wire EDM process [16]. Some researchers have used both Taguchi method and grey relational analysis simultaneously in their research work in turning [17, 18] and milling [19, 20].

Based on the above literature review, the present investigation has been concentrated on application of Taguchi's orthogonal array for planning the experimental work on milling operation. The

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controlling factors selected are speed, feed and depth of cut and four levels for each factor are selected. The grey relational analysis method is then applied to study the effect of the above controlling factors on the targeted factors viz. cutting force, torque and power. Thus an optimal parameter combination has been obtained by this combined approach.

2. Grey Relational Analysis

Grey relational analysis was proposed by Deng [13, 21, 22]. In grey relational analysis, the concept of black and white is used where black represents having no information and white represents having all information. A grey system has a level of information between black and white [23]. This analysis can be used to represent the grade of correlation between two sequences so that the distance of two factors can be measured discretely. In the case when experiments are ambiguous or when the experimental method cannot be carried out exactly, grey analysis helps to compensate for the shortcoming in statistical regression [24]. Grey relation analysis is an effective means of analyzing the relationship between sequences with less data and can analyze many factors that can overcome the disadvantages of statistical method [25].

The procedure of grey relational analysis has been laid down by Deng [13, 22]. The same has been used by many researchers in their investigations [18, 20, 26, 27]. In present work, more emphasis has been laid down on the grey relational analysis procedure laid down by Chornng Jyh Tzeng et al in their publication [18]. The same is briefly discussed in the coming sections.

2.1 Data normalization

Grey data normalization must be performed before Grey correlation coefficients (GRC) can be calculated. A series of various units must be transformed to be dimensionless. Usually, each series is normalized by dividing the data in the original series by their average.

Let the original reference sequence and sequence for comparison be represented as $x_0(k)$ and $x_i(k)$, $i=1, 2, \dots, m$; $k=1, 2, \dots, n$, respectively, where m is the total number of experiment to be considered, and n is the total number of observation data. Data preprocessing converts the original sequence to a comparable sequence. Several methodologies of preprocessing data can be used in Grey relation analysis, depending on the characteristics of the original sequence [18, 20]. If the target value of the original sequence is “the-larger-the-better”, then the original sequence is normalized using the following equation:

$$x_i^*(k) = \frac{x_i^{(0)}(k) - \min x_i^{(0)}(k)}{\max x_i^{(0)}(k) - \min x_i^{(0)}(k)} \quad (1)$$

If the purpose is “the-smaller-the-better”, then the original sequence is normalized as follows:

$$x_i^*(k) = \frac{\max x_i^{(0)}(k) - x_i^{(0)}(k)}{\max x_i^{(0)}(k) - \min x_i^{(0)}(k)} \quad (2)$$

However, if there is “a specific target value”, then the original sequence is normalized using,

$$x_i^*(k) = 1 - \frac{|x_i^{(0)}(k) - OB|}{\max\{\max x_i^{(0)}(k) - OB, OB - \min x_i^{(0)}(k)\}} \quad (3)$$

where OB is the target value.

Alternatively, the original sequence can be normalized using the simplest methodology that is the values of the original sequence can be divided by the first value of the sequence, $x_i^{(0)}(1)$.

$$x_i^*(k) = \frac{x_i^{(0)}(k)}{x_i^{(0)}(1)} \quad (4)$$

Where $x_i^{(0)}(k)$ is original sequence, $x_i^*(k)$ is the sequence after the data preprocessing, $\max x_i^{(0)}(k)$ is the largest value of $x_i^{(0)}(k)$, $\min x_i^{(0)}(k)$ is the smallest value of $x_i^{(0)}(k)$.

2.2 Grey relational coefficients and grades

After data preprocessing, a Grey relational coefficient (GRC) can be calculated using the preprocessed sequences. The Grey relational coefficient is obtained from the following equation:

$$\gamma(x_0^*(k), x_i^*(k)) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{0i}(k) + \zeta \Delta_{\max}} \quad (5)$$

$$0 < \gamma(x_0^*(k), x_i^*(k)) \leq 1 \text{ where}$$

$$\Delta_{0i}(k) = |x_0^*(k) - x_i^*(k)|$$

$$\Delta_{\max} = \max_{\forall j \in i} \max_{\forall k} |x_0^*(k) - x_i^*(k)|$$

$$\Delta_{\min} = \min_{\forall j \in i} \min_{\forall k} |x_0^*(k) - x_i^*(k)|$$

ζ is the distinguishing coefficient, $\zeta \in [0, 1]$

A grey relational grade is a weighted sum of the grey relational coefficients and is defined by the following equation:

$$\gamma(x_0^*, x_i^*) = \sum_{k=1}^n \beta_k \gamma(x_0^*(k), x_i^*(k))$$

$$\sum_{k=1}^n \beta_k = 1 \tag{6}$$

Here, the grey relational grade (GRG) $\gamma(x_0^*, x_i^*)$ represents the level of correlation between the reference and comparability sequences. If the two sequences are identical, then the value of the grey relational grade equals to one. The grey relational grade is also an indication of the degree of influence exerted by the comparability sequence on the reference sequence. Consequently, if a particular comparability sequence is more important to the reference sequence than other comparability sequences, the grey relational grade for that comparability sequence and the reference sequence will exceed that for other grey relational grades. The grey relational analysis is actually a measurement of the absolute value of data difference between the sequences and can be used to approximate the correlation between the sequences.

3. Experimental Setup and Procedure

3.1 Experimental setup

Experiments are performed to measure the cutting force using a 3D milling tool dynamometer. The forces along X, Y and Z axis are measured during plain milling operation. The experiment involved collection of force data while machining the flat surface on a piece of Mild Steel plate having dimension of 200 x 110 x 20 mm at various combinations of speed, feed and depth of cut. While force measurement, all three dimensional forces are measured viz. cutting force, feed force and radial force. In comparison to other two forces, cutting force being larger, is used for further investigation. While recording the forces machining is carried out for sufficient length of cut and readings are recorded. Torque and power are calculated from the experimental value of cutting force using the method described in literature [28]. The experiments are carried out on a column and knee type horizontal milling machine with a helical milling cutter. The experimental set up is shown in Fig. 1. Specifications of the machine and cutter are given in Table 1.

3.2 Taguchi's design of experiments

The DOE includes three controllable milling parameters (factors) at four levels, the values of which are tabulated in Table 2. For this specific combination of three parameters having values at four levels, G. Taguchi [31] has suggested to apply L16 Orthogonal array and accordingly L16 has been employed to explore

the process interrelationships within the experimental frame. Each parameter was assigned to a column, according to standard linear graph [31].



Fig. 1 Experimental Setup

Table 1: Machine & Cutter Specifications

Machine	Type: Column and knee type Speed range in steps (rpm): 60, 102, 148, 246, 386, 427, 619, 669, 1000 Feed range in steps (mm/min): 10, 16, 25, 46, 71, 111 Spindle arbor: 25.4 mm Spindle main motor: 3 HP Table feed motor: 0.5 HP
Milling Cutter	Type: helical External diameter: 77 mm Bore diameter: 25 mm Rake angle: 65° Number of teeth: 12 Helix angle: 30°

The orthogonal design of the parameters shown in Table 2 as well as the observations is mentioned in Table 3.

3.3 Application of GRA

The first step of GRA includes the linear normalization of the DOE data according to the type of performance response. It is desired that power consumption should be minimum so in the context of Taguchi methodology, cutting force, torque and power are considered lower-the-better performance responses. Hence normalization has to be carried out as per equation (2). The normalized values are shown in Table 4. The normalized values are ranged between zero and one; the larger values yield better performance and the ideal value should be equal to one, $x_0(k) = 1$.

Table 2: Parameters & Their Levels

Parameter (Factor)	Level 1	Level 2	Level 3	Level 4
A: Speed (S) (rpm)	60	148	386	669
B: Feed (f) (mm/min)	10	16	25	46
C: Depth of cut (d) (mm)	0.5	1	1.5	2

Table 3: L16 Orthogonal Array of Experimental Runs and Results

Run	Parameters			Response		
	A	B	C	Cutting Force (F _x) N	Torque (T) kN.mm	Power (P) kW
1	60	10	0.5	439.24	16.91	106.19
2	60	16	1	710.3	27.35	171.74
3	60	25	1.5	1182.27	45.52	285.85
4	60	46	2	2098.47	80.79	507.37
5	148	10	1	319.68	12.31	190.65
6	148	16	0.5	376.52	14.49	224.55
7	148	25	2	811.24	31.23	483.82
8	148	46	1.5	1129.94	43.5	673.89
9	386	10	1.5	207.07	7.97	322.09
10	386	16	2	285.77	11	444.5
11	386	25	0.5	278.52	10.72	433.22
12	386	46	1	552.92	21.29	860.04
13	669	10	2	123.58	4.76	333.15
14	669	16	1.5	189.73	7.3	511.48
15	669	25	1	243.24	9.36	655.74
16	669	46	0.5	354.86	13.66	956.65

The grey-relational coefficient (GRC) determines the relationship between the ideal and actual normalized response. The grey-relational coefficient (GRC) determines the relationship between ideal and actual normalized response. The value of GRC can be calculated using equation (5). The distinguishing coefficient ζ is set between zero and one. The purpose of distinguishing coefficient is to expand or compress the range of grey relational coefficient. Y. Kuo et al [29] has found that no matter what the distinguishing coefficient is, the ranking order remains the same. Later Sood et al [30] has also

concluded the same based on sensitivity analysis of their results. In present case, it is taken as 0.5. In the last GRA step the grey-relational grades (GRG) are calculated by averaging the values of GRC for each performance response. Table 4 also shows the values of $\Delta_{0i}(k)$, grey relational coefficients (GRC) and Grey relational grade (GRG). This investigation employs the response table of the Taguchi method to calculate the average Grey relational grades for each factor level, as illustrated in Table 5. The response diagram is shown in Fig. 2.

Table 4: Calculated Grey Relational Coefficients and Grey Relational Grades for 16 Comparability Sequences

Exp. run	Normalized Reference Sequence			$\Delta_i(k) = x_0^*(k) - x_i^*(k) $			Grey relational coefficient			Grey Relational Grade
	Fx	T	P	Fx	T	P	Fx	T	P	
	1	1	1	Fx	T	P	Fx	T	P	
1	0.8402	0.8402	1	0.1598	0.1598	0	0.7578	0.7578	1	0.8385
2	0.7029	0.7029	0.9229	0.2971	0.2971	0.0771	0.6273	0.6273	0.866	0.707
3	0.4639	0.4639	0.7888	0.5361	0.5361	0.2112	0.4826	0.4826	0.703	0.5561
4	0	0	0.5283	1	1	0.4717	0.3333	0.3333	0.515	0.3937
5	0.9007	0.9007	0.9007	0.0993	0.0993	0.0993	0.8343	0.8343	0.834	0.8343
6	0.8719	0.8719	0.8608	0.1281	0.1281	0.1392	0.7961	0.7961	0.782	0.7915
7	0.6518	0.6518	0.5559	0.3482	0.3482	0.4441	0.5895	0.5895	0.53	0.5695
8	0.4904	0.4904	0.3325	0.5096	0.5096	0.6675	0.4952	0.4952	0.428	0.4729
9	0.9577	0.9577	0.7461	0.0423	0.0423	0.2539	0.9219	0.9219	0.663	0.8357
10	0.9179	0.9179	0.6022	0.0821	0.0821	0.3978	0.8589	0.8589	0.557	0.7582
11	0.9215	0.9215	0.6155	0.0785	0.0785	0.3845	0.8643	0.8643	0.565	0.7646
12	0.7826	0.7826	0.1136	0.2174	0.2174	0.8864	0.6969	0.6969	0.361	0.5848
13	1	1	0.7331	0	0	0.2669	1	1	0.652	0.8839
14	0.9665	0.9665	0.5235	0.0335	0.0335	0.4765	0.9372	0.9372	0.512	0.7955
15	0.9394	0.9394	0.3538	0.0606	0.0606	0.6462	0.8919	0.8919	0.436	0.74
16	0.8829	0.8829	0	0.1171	0.1171	1	0.8102	0.8102	0.333	0.6512

Table 5: The Response Table for GRG

Levels	Parameters		
	A	B	C
1	0.8318	1.1308	1.0153
2	0.8894	1.0174	0.9554
3	0.9256	0.8767	0.8867
4	1.0236	0.7009	0.8685

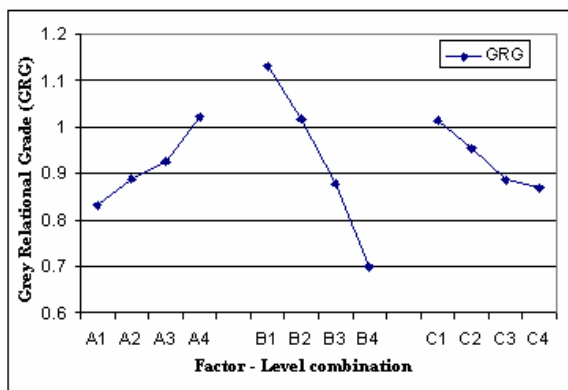


Fig. 2 Response Diagram for Grey Relational Grades

Since the Grey relational grades represented the level of correlation between the reference and the comparability sequences, the larger Grey relational grade means the comparability sequence exhibiting a stronger correlation with the reference sequence. Based on this study, one can select a combination of the levels that provide the largest average response. In Table 5, the combination of A4, B1 and C1 shows the largest value of the Grey relational grade for the factors A, B and C, respectively. Therefore, the combination A4 - B1 - C1 with a speed of 669 rpm, a feed of 10 mm/min and depth of cut of 0.5 mm is the optimal parameter combination of the current operation under study where it is desired to reduce the force and hence the power consumption. It

should be noted at the point that MRR and surface finish are also important parameters involved in this process. Consideration of these parameters may affect the results obtained here. Cost depends not only on power but other parameters like MRR. Increase in MRR can reduce the cost and this is possible at larger feed and depth of cut. However, increase in MRR may result in deterioration of surface roughness. Hence, considering MRR and/or surface roughness along with power may result in different optimum combination.

Further, the analysis of variance (ANOVA) [10, 11, 32] is carried out for input factors using the calculated values of Grey relational grade of Table 5 and the response table of Table 5. The results of ANOVA are shown in Table 6.

Table 6: ANOVA Results

Factor	SS	DF	MS	F	%P
A	0.8452	2	0.4226	1	27.73
B	1.281	2	0.6405	1.516	42.02
C	0.922	2	0.461	1.091	30.25
Total	3.0482				100

According to Table 6, the factor B, the feed with 42.02% of contribution, is the most significant controlled parameters for the current operation; the depth of cut has 30.25% contribution, while the speed has 27.73% of contribution.

5. Conclusions

The Grey relational analysis based on an orthogonal array of the Taguchi method is found to be an effective way of optimizing the milling operations. The analytical results are summarized as follows:

1. From the response table of the average Grey relational grade, it is found that the largest value of the Grey relational grade for the speed of 669 rpm, the feed rate of 10mm/min and the depth of cutting of 0.5mm. It is the recommended levels of the controllable parameters of the milling operation when the minimization of the force, torque and power are considered.
2. Through ANOVA, it is found that feed, depth of cut and speed are the most significant parameters that have 42.02%, 30.25% and 27.73% contribution respectively on the milling process when minimization of force, torque and power are simultaneously considered.

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Nomenclature

Symbol	Meaning
$x_0(k)$	Original reference sequence
$x_i(k)$	Sequence for comparison
m	Total number of experiment to be considered
n	Total number of observation data.
$x_i^{(0)}(k)$	Original Sequence
$x_i^*(k)$	Sequence after data pre processing
max.	Largest value of $x_i^{(0)}(k)$
$x_i^{(0)}(k)$	Smallest value of $x_i^{(0)}(k)$
min.	
$x_i^{(0)}(k)$	
ζ	Distinguishing coefficient
GRC	Grey relational coefficient
GRG	Grey relational grade
S	Speed
f	Feed
d	Depth of cut