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KINEMATIC MODELING AND SIMULATION OF SPINDLE ERRORS IN A MINIATURIZED MACHINE TOOL

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ABSTRACT

Spindle rotation accuracy is one of the important items that affect the machining performance of a miniaturized machine tool. Capacitive sensor based measurement technique is commonly used for evaluating error motions of machine tool spindle using a master cylinder. This paper presents a kinematic model for analyzing error motions of miniaturized machine tool spindle. Principles of rigid body kinematics and homogeneous transformation matrices are used for developing the model. In the present work, fourier series is used for modeling the synchronous components of spindle error motion and form error of master cylinder. Asynchronous spindle error motion is characterized using normal probability distribution. Spindle radial error data of a miniaturized machine tool is simulated using the proposed model and compared with experimental results. Simulation results proved that the proposed model is useful in accurate interpretation and analysis of spindle error measurement.

Keywords: *Kinematic Model, Spindle Error Motions, Simulation*

1. Introduction

Spindle is a key component of the miniaturized machine tool that positions and transmits power to a micro cutting tool or a work-piece. Precise axis of rotation of the spindle is essential to produce high precision parts. Machine tool error sources like manufacturing imperfections, form error of the bearing surface, thermal growth due to machining and lack of kinematic arrangements cause kinematic deviations in axis of rotation and it is referred as spindle error motions. Fig.1 shows a geometric model of a typical spindle system of a machine tool and its axis of rotation.

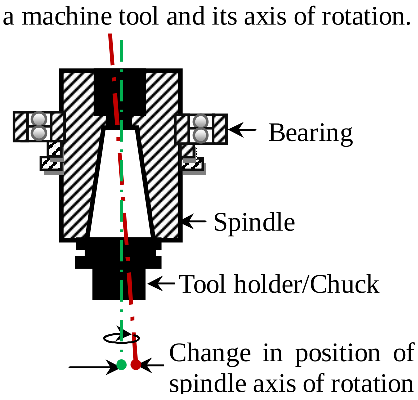

Fig. 1 A Typical Machine Tool Spindle

Spindle error motions include synchronous, asynchronous components and it causes tool positioning errors while machining the work piece. Synchronous error motion components are repeatable and periodic components in each spindle revolution. Asynchronous components are the non periodic components of spindle radial error motion. Spindle running error has a direct influence on both geometric shapes and surface roughness of the work pieces produced by machining operations involving rotary motions [1]. Consequently, evaluation of accuracy of axis of rotation of spindle is essential to ensure the positioning accuracy of a machine tool.

Spindle error analysis enables measurement of relative change in location between the tool and work piece that causes degradation in surface finish, roundness feature size and feature location [2]. An effort has been carried out to unify the specifications, terminologies and testing methods related to axes of rotation of machine tools [3]. ANSI /ASME B 89.3.4M [4] laid a modern foundation for understanding, specifying and testing the error motion concept of axes of rotation. Principle and operation of magnetic ball bar for testing two or three dimensional accuracy of a machine tool was described [5]. Chapman developed a capacitance based ultra precision spindle error analyzer for measuring spindle error motions [6]. Lee and Wi presented a technique for measuring three dimensional positioning accuracy of a spindle using capacitance sensors and master ball [7].

Accuracy of spindle error measurement is limited by centering error, surface profile error of the master. Digital filtering technique was proposed for removing the contribution of centering error from the measurement data [8]. Donaldson developed a reversal

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method for separating the form error of artifact from the radial error motion data using polar profile averaging method [9]. Alternate approaches like dual probe and dual orientation methods were proposed for separating the profile error of master from spindle error motion [10]. The volumetric error model was modified to account spindle error motions of a machine tool for predicting roundness error of the part [11]. A kinematic spindle with exact constraints was fabricated and an analytical model was derived to predict radial error motions of the spindle using roundness profile of rotor [12]. A new method was presented for measuring roundness error and spindle error using four capacitance probes [13]. Improved implementation of Donaldson reversal method and frequency domain low pass filtering techniques were demonstrated for nanometer level evaluation of spindle error motion [14].

New methods were attempted to improve the accuracy of spindle error motion measurement and overcome the undesirable measurement errors caused by the reference master. A new spindle error measurement system consisting of rotational fixture with a built in laser diode and position sensitive detectors was developed [15]. Laser diode and quadrant sensors were used to develop a measurement system to determine spindle error motions at high speeds [16]. Laser interferometer is used for evaluating radial and axial error motion of a machine tool [17].

It is observed that very few attempts have been carried out in developing a model based approach for the analysis of spindle error motion. In this work, kinematic model is presented to analyze and indentify the contribution of centering error and form error in spindle error measurement. Synchronous and asynchronous components of spindle error data are characterized using a fourier series model and normal probability distribution respectively. Experimental and simulation results of the proposed model are presented in this paper.

2. Modeling of Spindle Error Motions

In practice, it is impossible to determine the axis of rotation of spindle and analyze its error motions. Hence a spindle coordinate frame ($X_sY_sZ_s$) and a reference coordinate frame (XYZ) are formulated for determining error motions during spindle rotation. Spindle error motions contain independent translation motions (δ_x δ_y δ_z) and tilt motions (α,β) with respect to the reference coordinate frame XYZ as shown in Fig. 2.

Master cylinder of known dimension is mounted in the spindle. Capacitive sensors are positioned to formulate reference coordinate system XYZ and it is assumed to be aligned with axis of rotation of spindle. Schematic arrangement of spindle error motion measurement is shown in Fig.3.

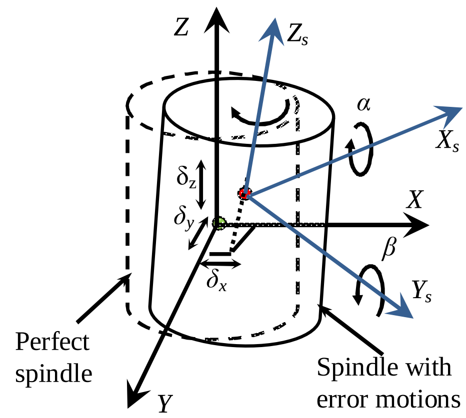


Fig. 2 Spindle Error Motions

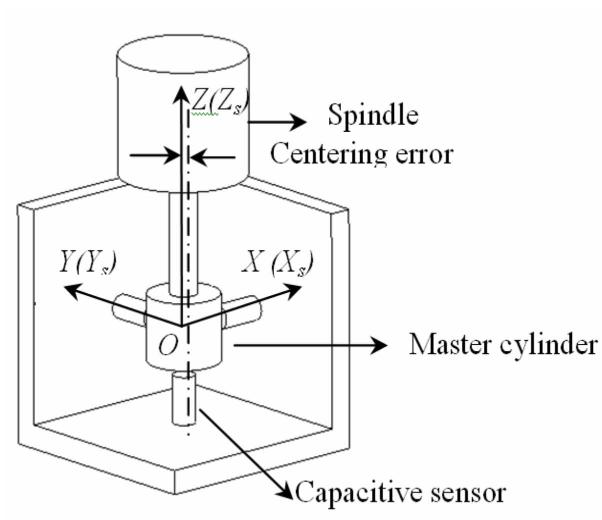


Fig. 3 Schematic Arrangement of Capacitive Sensors for Measuring Spindle Error Motions

Capacitive sensor measures the displacement of master cylinder at time t_i and it includes the contribution of centering error, form error of master cylinder. In this work, a mathematical model is derived for displacement of the master cylinder during spindle rotation.

2.1 Modeling the position of master cylinder

Fig.4 shows the geometric arrangement of master cylinder and capacitive sensor for measuring spindle error motions. G is the geometric center of master cylinder and O is the axis of rotation of spindle. Centering error of master cylinder in X, Y directions is denoted as e_x, e_y respectively. Surface profile of the master cylinder deviates from perfect circle due to form error and its contribution is given by Δr_x, Δr_y.

Let (x, y, z) denote the mean position of master cylinder from the capacitive sensors when the output of capacitive sensors are adjusted to zero. At any instant of time t_i, the displacement of master cylinder with

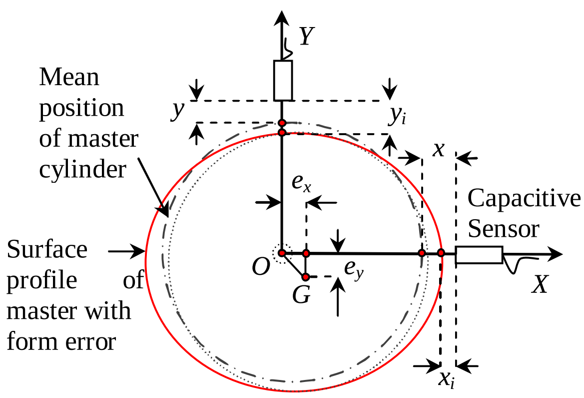


Fig. 4 Change in Position of Master Cylinder

the capacitive sensor is given by (x_i, y_i, z_i), as shown in Fig.4. The displacement of master in *Z* direction will be perpendicular to *XY* plane and it is not shown in Fig.4. Instantaneous position of master cylinder is modeled using homogeneous transformation matrix given by equation (1).

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = [T_s][T_r][T_c] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (1)$$

In equation (1), *[T$_r$]* represents transformation matrix as given for the ideal rotation of master cylinder which includes form error in its surface profile. *[T$_c$]* is the transformation matrix for representing the contribution of centering error of master cylinder. As the centering error is introduced during mounting of the master in the stationary spindle, its contribution $[T_c]$ precedes ideal rotation of the spindle. *[T$_s$]* corresponds to independent translation motions (δ_{xi}, δ_{yi}, δ_{zi}) and tilt error motions (α_i and β_i) of spindle at time t_i and it is given by equation (2)

$$[T_S] = \begin{bmatrix} 1 & 0 & -\beta_i & \delta_{xi} \\ 0 & 1 & \alpha_i & \delta_{yi} \\ -\alpha_i & \beta_i & 1 & \delta_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

In equation (1), *[T$_r$]* corresponds to the transformation matrix for representing ideal rotation of spindle using a master and it is given by equation (3). In equation (3), θ_i represents the angular position of spindle at time t_i. $\Delta r_{xi}, \Delta r_{yi}$ represents the contribution of form error of master in *X, Y* directions.

$$[T_r] = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & \Delta r_{xi} \\ \sin(\theta_i) & \cos(\theta_i) & 0 & \Delta r_{yi} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

Misalignment of master cylinder with axis of rotation of spindle in *X, Y* direction is given by e_x, e_y respectively and it is represented by equation (4).

$$[T_c] = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

Substituting Equations (2), (3), (4) in Equation (1).

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\beta_i & \delta_{xi} \\ 0 & 1 & \alpha_i & \delta_{yi} \\ -\alpha_i & \beta_i & 1 & \delta_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & \Delta r_{xi} \\ \sin(\theta_i) & \cos(\theta_i) & 0 & \Delta r_{yi} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (5)$$

Multiplying the above matrices in proper order, the kinematic transformation matrix for spindle error measurement is derived as given below:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & -\beta_i & \delta_{xi} + \Delta r_{xi} + e_x\cos(\theta_i) - e_y\sin(\theta_i) - \beta_i z \\ \sin(\theta_i) & \cos(\theta_i) & \alpha_i & \delta_{yi} + \Delta r_{yi} + e_x\sin(\theta_i) + e_y\cos(\theta_i) + \alpha_i z \\ \alpha_i\sin(\theta_i) - \beta_i\cos(\theta_i) & \alpha_i\cos(\theta_i) + \beta_i\sin(\theta_i) & 1 & \beta_i(\Delta r_{yi}) - \alpha_i(\Delta r_{xi}) + \delta_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad (6)$$

Change in position of master cylinder surface $(\Delta x_i, \Delta y_i, \Delta z_i)$ in *X, Y, Z* directions can be obtained by extracting the translational sub matrix as given by equation (7), (8), (9).

$$\Delta x_i = \delta_{xi} + \Delta r_{xi} + e_x\cos(\theta_i) - e_y\sin(\theta_i) - \beta_i z \qquad (7)$$

$$\Delta y_i = \delta_{yi} + \Delta r_{yi} + e_x\sin(\theta_i) + e_y\cos(\theta_i) + \alpha_i z \qquad (8)$$

$$\Delta z_i = \beta_i(\Delta r_{yi}) - \alpha_i(\Delta r_{xi}) + \delta_{zi} \qquad (9)$$

Equation (7), (8), (9) represent the spindle error data obtained from the capacitive sensors in X, Y, Z directions. The equations contain the contributions of radial, axial, tilt error motions of spindle along with centering error, form error of master cylinder.

2.2 Modeling spindle error motions and form error of master cylinder

In the miniaturized machine tool, analysis of radial error motions are critically important than the axial, tilt error motion as the depth of cut of micro cutting tool is less. In equation (7) and (8), δ_{xi}, δ_{yi} represent the radial error motion of spindle and it contains synchronous and asynchronous components. In this work, fourier series is used for expressing the synchronous components of radial error motion. Asynchronous components are assumed to follow normal random distribution with zero mean and constant variance. Equations (10) and (11) are formulated for describing the radial error motion of the spindle with synchronous and asynchronous components in *X, Y* directions

$$\delta_{xi} = \sum_{h=2}^{H_b} B_h \cos(h\theta_i + \phi_h) + \varepsilon_i \tag{10}$$

$$\delta_{yi} = \sum_{h=2}^{H_b} B_h \sin(h\theta_i + \phi_h) + \varepsilon_i \tag{11}$$

In equation (10) and (11), H_b represents number of harmonics (Cycles per revolution) required for representing synchronous spindle error motion. B_h, ϕ_h expresses amplitude and phase of harmonic components of spindle error motion. ϵ_i denotes the asynchronous components of spindle radial error motion.

Form error of the master cylinder Δr_{xi}, Δr_{yi} repeats for each revolution of the spindle, hence its contribution is contained in the synchronous components of spindle error measurement. Hence similar expressions shown in Equation (10) and Equation (11) are used for modeling the contribution of systematic form error of master cylinder in *X, Y* directions and it is given by Equation (12) and (13).

$$\Delta r_{xi} = \sum_{h=1}^{H_a} A_h \cos(hw\theta_i + \varphi_h) \tag{12}$$

$$\Delta r_{yi} = \sum_{h=1}^{H_a} A_h \sin(hw\theta_i + \varphi_h) \tag{13}$$

In equation (12) and (13), w is the integer number for representing the fundamental spatial frequency of surface profile in cycles per revolution. H_a represent number of harmonics in the form error of the master. A_h, φ_h corresponds to the amplitude and phase of harmonic components of form error of master.

2.3 Simplified model for spindle error data

Equation (7), (8) is simplified by substituting $e_x = E\cos(\varphi)$; $e_y = E\sin(\varphi)$ and the equations (10), (11), (12), (13).

$$\Delta x_i = C + \sum_{h=2}^{H_b} B_h \cos(h\theta_i + \phi_h) + \sum_{h=1}^{H_a} A_h \cos(hw\theta_i + \varphi_h) + E\cos(\theta_i + \varphi) - \beta_i z + \varepsilon_i \tag{16}$$

$$\Delta y_i = C + \sum_{h=2}^{H_b} B_h \sin(h\theta_i + \phi_h) + \sum_{h=1}^{H_a} A_h \sin(hw\theta_i + \varphi_h) + E\sin(\theta_i + \varphi) + \alpha_i z + \varepsilon_i \tag{17}$$

A constant term C is also included in the model for representing offset between the mean position of master cylinder and standoff distance of capacitive sensor. Trigonometric addition formulas are used for simplifying equations (16) and (17) for the harmonic cutoff H in cycle per revolution (CPR).

$$\Delta x_i = C + \sum_{h=1}^{H} C_h \sin(h\theta_i + \gamma_h) - \beta_i z + \varepsilon_i \tag{18}$$

$$\Delta y_i = C + \sum_{h=1}^{H} C_h \cos(h\theta_i + \gamma_h) + \alpha_i z + \varepsilon_i \tag{19}$$

Equation (18), (19) is suitable for analyzing spindle radial error data at the discrete angular position of spindle. Discrete time interval for sampling the spindle error data is calculated using equation (20) for the given sampling frequency.

$$\Delta t = 1/f_s \tag{20}$$

Angular position of spindle at the discrete time t_i can be written as a function spindle rotational frequency (f_0)

$$\theta_i = 2\pi f_0 t_i \tag{21}$$

Substituting Equation (21) in Equation (18) and (19).

$$\Delta x_i = C + \sum_{h=1}^{H} C_h \sin(2\pi h f_0 t_i + \gamma_h) - \beta_i z + \varepsilon_i \tag{22}$$

$$\Delta y_i = C + \sum_{h=1}^{H} C_h \cos(2\pi h f_0 t_i + \gamma_h) + \alpha_i z + \varepsilon_i \tag{23}$$

In Equation (22) and (23), the sum of trigonometric functions represents the synchronous components of spindle error data and it contains the centering error and form error of master cylinder. C_h is the harmonic amplitudes of spindle error data. ϵ_i represent the asynchronous components of spindle error data. t_i is the discrete sampling time of spindle error measurement.

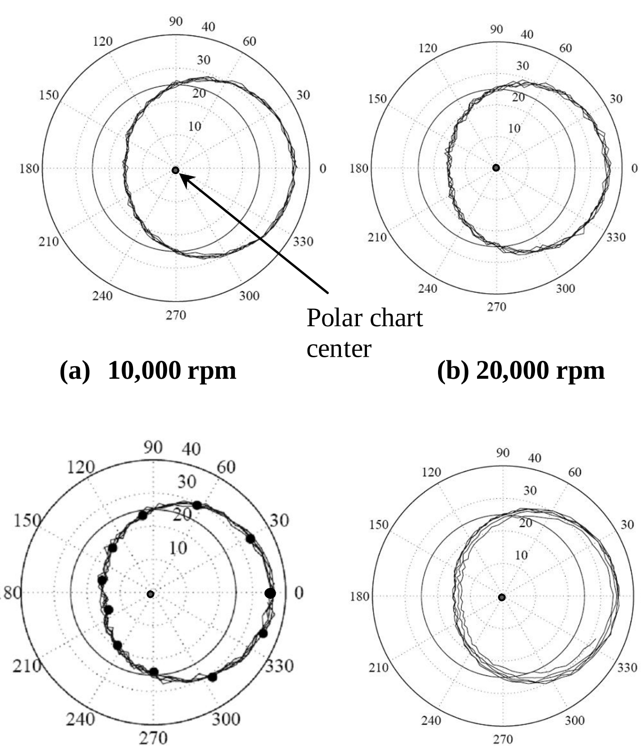
3. Simulation of Spindle Error Data

In order to demonstrate the performance of the developed model, spindle error data is simulated for various spindle speeds using equation (21). Tilt and axial error motions are assumed to be negligible ($\beta_i =0, \Delta z =0$). Typical values of model parameters are assumed for Table.1. For simplicity, radial error data is simulated for 10 harmonic cutoff and phase values are assumed to be zero. Array of normal distribution random numbers are generated for specified variance values for generating asynchronous components of spindle error data.

Table 1: Values Used for Simulation of Spindle Error Data

S.No	Model parameters	Values
1	Spindle speed(rpm)	10000,20000 50000,60000
2	Number of harmonics	10
3	Amplitude of harmonics (C_h)	10.00,0.150,0.1,0.050, 0.030, 0.015,0.010,0.020 0.025,0.040
4	Variance of asynchronous (ϵ_i)	0.1 -0.2 μm^2

Fig.5 shows the polar plot for the samples of simulated data in the base circle radius of 25μm at various spindle speeds. Simulated data shows the deviation in polar profile center from polar chart center due to centering error of master cylinder. Samples values of simulated radial error data for the spindle speed of 50,000 rpm is given in Table.2 and it is shown as black dots in Fig.5(c).



(a) 10,000 rpm (b) 20,000 rpm

(c) 50,000 rpm (d) 60,000 rpm

Fig. 5 Samples of Simulated Spindle Error Data Obtained at the Various Spindle Speeds

Table 2: Samples of Simulated Radial Error Data Obtained for the Spindle Speed of 50,000 pm

S.No	Sampling time (t_i) (msec)	Angular position of spindle (θ_i)(deg)	Simulated data (Δx_i)	
			Raw data	In 25μm base circle
1	0.000	0.000	10.485	35.485
2	0.096	28.766	8.508	33.508
3	0.212	63.294	4.442	29.442
4	0.327	97.812	-1.533	23.467
5	0.442	132.321	-6.814	18.186
6	0.558	166.863	-9.104	15.896
7	0.673	201.369	-10.505	14.495
8	0.788	235.880	-5.695	19.305
9	0.904	270.405	-0.936	24.064
10	1.019	304.927	6.262	31.262
11	1.135	339.477	10.416	35.416

As the speed increases, the variance of asynchronous components are reduced, hence the simulated data at 60,000 rpm are smoother than the other spindle speeds.

4. Measurement of Spindle Error Motion of Miniaturized Machine Tool

In this work, single point asynchronous motion (SPAM) test [4] is conducted to measure the radial error motions of miniaturized machine tool spindle. Fig.6 shows the experimental arrangement with a master cylinder and capacitive sensor positioned in the radial direction of the miniaturized machine tool spindle.

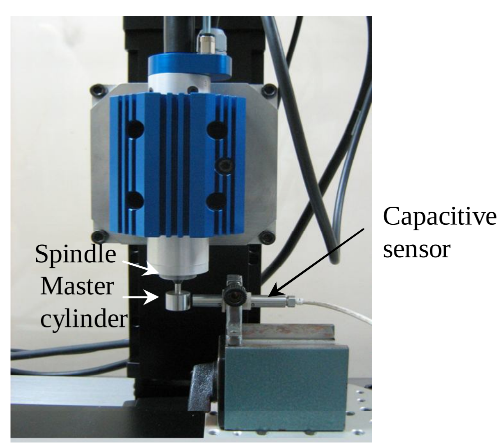
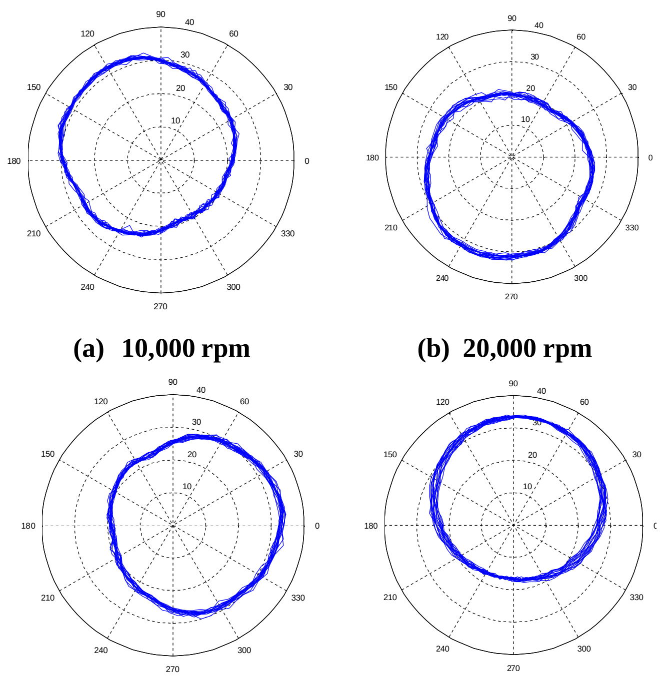


Fig. 6 Experimental Arrangement for Radial Error Motion Measurement of Miniaturized

Machine Tool Spindle

A computer aided data acquisition system is used for acquiring the spindle error motions for the given sampling frequency. Fig.7 shows the samples of radial error data obtained at various spindle speeds for 16 spindle revolutions.



(a) 10,000 rpm **(b) 20,000 rpm**

(b) 50,000 rpm **(d) 60,000 rpm**

Fig. 7 Samples of Experimental Spindle Radial Error Data Measured at Various Spindle Speeds.

It can be seen that polar profile of measurement data deviates from the polar chart center due to the effect of centering error as seen from Fig.5(a) to (d). However, there exists a phase error between the simulated profile and experimental data. It is due to different orientation of centering error of master cylinder and spindle speed variations during spindle error measurement.

4.1 Correlation of proposed model to the measured spindle error data

Proposed model is fitted using least squares method [18] to the measured data for 10 harmonic cutoff. Results are shown in Fig.8 for the measured data obtained at spindle speed of 50,000 rpm. A correlation coefficient of 0.981 was found between the proposed model and the experimental data.

This proves the effectiveness of proposed approach for analyzing synchronous and asynchronous components of spindle error measurement.

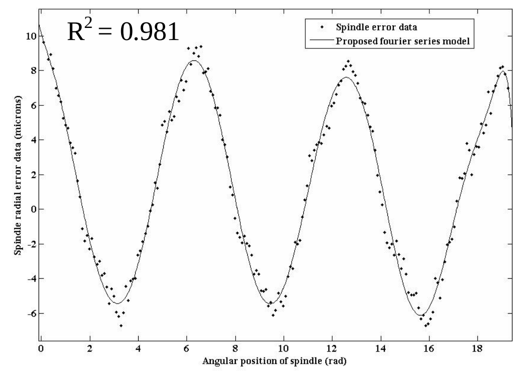


Fig. 8 Correlation of Proposed Model to the Measured Spindle Error Data

5. Conclusion

This paper describes a kinematic modeling approach for capacitive sensor based measurement and analysis of spindle errors of a miniaturized machine tool. Following conclusions were drawn from the simulation and experiment results of the proposed approach.

(a) Proposed kinematic modeling method provides simple approach for analyzing contributions of centering error and form error in spindle error measurement.

(b) Correlation coefficient of 0.981 of proposed model to the spindle error data is found to be satisfactory for analyzing spindle radial error measurement.

(c) The developed model will be useful in experimental investigations on dynamic behavior of a spindle and assessing its performance on accurate tool positioning during machining.

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Nomenclature

Symbol	Meaning	Unit
A_h	Amplitude values of the harmonic components of master cylinder surface	µm
B_h	Amplitude values of the harmonic components of spindle error motion	µm
C_h	Amplitude values of the harmonic components of spindle error data	µm
e_x, e_y	Centering error in X,Y directions	µm
H	Harmonic cutoff for synchronous components of spindle error data	CPR
H_a	Number of harmonics for representing form error of master cylinder	CPR
H_b	Number of harmonics for representing synchronous spindle error motion	CPR
t_i	Sampling time	Sec
w	Fundamental frequency of surface profile of master cylinder surface	CPR
(x, y, z)	Mean position of master cylinder	µm
(x_i, y_i, z_i)	Position of master cylinder at time t_i	µm
α_i, β_i	Tilt error motions in X,Y directions	rad
δ_{xi}, δ_{yi}	Radial error motions in X,Y directions	µm
δ_{zi}	Axial error motion in Z direction	µm
Δr_{xi}, Δr_{yi}	Contribution of form error in the spindle error data	µm
$(\Delta x_i, \Delta y_i, \Delta z_i)$	Change in position of master cylinder in X,Y,Z directions	rad
θ_i	Angular position of spindle	rad
$ø_h$	Phase of harmonic components of spindle error motion	rad
φ_h	Phase of harmonic components of master cylinder surface profile	rad
γ_h	Phase of harmonic components of spindle error data	rad
ϵ_i	Asynchronous components	µm