Journal of Manufacturing Engineering, September 2019, Vol. 14, Issue. 3, pp 106-110

A NEW MODEL FOR COUPLING OF TRIBOLOGICAL AND MECHANICAL MODELS OF THIN STRIP AND FOIL ROLLING

https://doi.org/10.37255/jme.v4i3pp106-110

***Manoj Kumar[1], Jyoti Raman[2] and Priya[2]**

[1] Department of Mechanical Engineering, Echelon Institute of Technology Faridabad, Haryana-121101, India
2Maa Yashodhara Private Industrial Training Institute Sambhuganj, Banka, Bihar -813211, India

ABSTRACT

A new analysis for cold rolling of thin strip and foil is developed. This model follows the approach of Fleck et al [8], but relaxes their assumption of a central flat neutral zone. Instead of following their inverse method to obtain the pressure distribution in this neutral zone, an explicit equation for the contact pressure variation is obtained from the sticking condition in this region. This significantly simplifies the solution method, leading to a much more robust algorithm. Moreover, the method treats the cases either where the roll retains its circular arc or where there is very significant roll deformation in the same way, greatly simplifying the method of obtaining solutions. This will facilitate the incorporation of other effects such as the friction models currently being developed. Results are in line with the theory of Fleck et al [8]. The effect of entry and exit tensions on the non-dimensional load and forward slip is investigated. It is found that the effect of equal entry and exit tensions is equivalent to reducing the yield stress of the strip by this tension stress.

Keywords: *Metal rolling process, Thin strip and foil, Pressure distribution.*

1. Introduction

Modelling of thin strip and foil rolling is of great interest to industry due to the large tonnage of such material consumed each year. Modern set-up and control algorithms for rolling mills rely on robust and accurate mathematical models of the roll bite which can predict key parameters such as load, torque and forward slip as a function of the rolling parameters. These models either neglect roll deformation or allow for this by assuming an increased roll radius, for example using the well-known Hitchcock formula [1]. This model is improved by Jortner et al [2] to allow for a non-circular roll shape. Unfortunately he maintains the assumption that there is relative slip between the roll and the strip throughout the bite, except at the ‘neutral point’ where the relative slip between the roll and strip changes sign. These theories are successful in thick strip rolling, but unsatisfactory when applied to thin strip where there is significant roll elasticity.

An investigation by Johnson and Bentall [3] suggests that the plastic reduction of thin strip between two rolls occurs in two zones separated by an extensive no-slip neutral zone. In this region the frictional traction falls below that for slipping friction. Based on this suggestion, a new theory of cold rolling thin foil is developed by Fleck and Johnson [4]. This is improved in the model of Fleck et al [5]. Here, deformation of the rolls is treated by modelling these as elastic half-spaces. The contact length is split into a series of zones, according to whether the strip is plastic or elastic and whether there is slip between the roll and strip. For the slipping regions, equilibrium of the strip is used to find the variation of pressure with rolling direction. For the no-slip neutral zone, which is taken as flat, a matrix equation is assembled which relates the elastic roll deformation to the normal pressure. This is inverted to find the pressure distribution in this region. To meet the continuity conditions at the boundaries between each of the zones, the positions of these boundaries are found using a Newton-Raphson scheme. Theoretical predictions for the roll shape in the bite using this model are in good agreement with experiments by Sutcliffe and Rayner [6]. This model is extended by Yuen and co-workers [7] to include strain hardening of the strip and by Domanti et al [8-9], modelling roll elasticity using the influence functions for circular rolls described by Jortner [2].

Although the models described in the previous paragraph have gained widespread support, both from industrialists and academics, they suffer from two major drawbacks. Firstly the principle of separating the bite into several zones, for which the boundaries have to be solved, is numerically unstable and time-consuming. Secondly the nature of the solution (e.g. what zones are needed) has to be identified before solving the problem. These deficiencies need to be overcome before friction modelling, which plays a key role in foil rolling, can be successfully coupled into the problem. An alternative

**Corresponding Author - E- mail: kumarm1968@rediffmail.com*

strategy which overcomes these difficulties is described by Gratacos et al [10], who define a friction law which simulates sticking friction in the neutral zone and slipping friction elsewhere. This strategy has also been used in a recent model which couples tribological and mechanical models of foil rolling [1]. The approach described in this paper takes its inspiration for modelling of the neutral zone from this method, although the details of the formulation are quite different. Elsewhere the model follows the theory by Fleck et al [5]. The theory and numerical scheme are described in sections 2 and 3, while results are presented in section 4.

2. Theory

Most of the theory is taken directly from the work by Fleck et al [5]. This is summarised below for completeness. The new element for modelling of the neutral zone is described in detail in section 2.5.

2.1. Equilibrium of slab element

Consider a slab element of the strip, as shown in Figure 1(a). Equilibrium gives

$$t\frac{d\sigma_x}{dx}+(\sigma+p)\frac{dt}{dx}+2q=0 \tag{1}$$

where x is the rolling distance, t is the strip thickness, averaged through the thickness of the strip, p is the interface pressure and q is the shear stress.

2.2 Roll shape

The variation of strip thickness *t* through the bite is given by

$$t=t_c+2b(x) \tag{2}$$

where $t_c=t_1-\frac{x_a^2-x^2}{R}$ is the strip thickness variation with an initial strip thickness t_1 corresponding to the circular arc for the undeformed roll. Here x_a is the value of x at the entry to the bite. b(x) is the elastic displacement of the roll in the vertical z direction at a point x due to the applied normal pressure distribution p, measured relative to the displacement at the entry position. Using the half-space solution described by Johnson [11], b(x) is given by

$$b(x)=-\frac{2}{\pi E_R^*}\int_{-x_a}^{x_a}p(s)\ln\left|\frac{s+x_a}{s-x}\right|ds \tag{3}$$

2.3 Elastic slip at entry and exit

The vertical elastic strain of the strip is given by:

$$\varepsilon_S(z)=-\frac{1}{E_S^*}\left(p(x)+\frac{\nu_S}{1-\nu_S}\sigma(x)\right) \tag{4}$$

Since there are no plastic strains in these regions, the rate of change of strip thickness due to the change in elastic strain in the rolling direction is equal to the roll slope dt/dx, giving

$$\frac{dp}{dx}+\frac{\nu_S}{1-\nu_S}\frac{d\sigma}{dx}=-\frac{E_S^*}{t}\frac{dt}{dx} \tag{5}$$

In practice we can neglect the last term which is small compared to the first term on the right hand side of the equation, to get:

$$\frac{dp}{dx}\cong-\frac{E_S^*}{t}\frac{dt}{dx}+\frac{\nu_S}{1-\nu_S}\frac{2q}{t} \tag{6}$$

Assuming a Coulomb friction law, the shear stress in the slipping elastic entry and exit regions is given by

$$q=\pm\mu p \tag{7}$$

where the positive sign is used for the backward slip at entry and the negative sign for the forward slip at exit. Alternative formulations incorporating a limiting shear stress or a friction factor approach could straightforwardly be used in the model.

2.4 Plastic slip

In the plastic region, a Tresca yield criterion is assumed.

$$p(x)+\sigma(x)=Y_S \tag{8}$$

Substituting this equation into Eq.1 gives

$$\frac{dp}{dx}=\frac{Y_S}{t}\frac{dt}{dx}+\frac{2q}{t} \tag{9}$$

In the plastic slip regions, the Coulomb friction law, Eq. 8, is again used.

2.5 Sticking region

Equations for the change in roll and strip strains in the sticking zone can be combined with flow continuity for the strip through this zone to derive expressions relating the pressure gradient and shear stress in this zone to the roll slope here. Results are summarised below and described in detail in the Appendix. In the plastic no-slip region, the pressure gradient and shear stress are given by:

$$\frac{dp}{dx}=-\frac{C_1E_S^*}{t}\frac{dt}{dx} \tag{10}$$

$$q=-\frac{C_1E_S^*}{2}\frac{dt}{dx} \tag{11}$$

For aluminium strip and steel rolls, $E_S^*\cong\frac{1}{3}E_R^*$, so that $C_1=1.05$. The pressure gradient and the shear stress in the elastic unloading region are given by:

$$\frac{dp}{dx}=-\frac{C_2E_S^*}{t}\frac{dt}{dx} \tag{12}$$

$$q = -\frac{C_2 C_3 E_S^*}{2}\frac{dt}{dx} \quad (13)$$

For aluminium strip and steel rolls, $C_2 = 1.36$, $C_3 = 0.62$. Comparing these equations with equations 11 and 12 for the plastic no-slip region, we find that they differ only by having slightly different coefficients. Equations 12 and 14 show that the pressure gradient is proportional to the roll slope in the sticking region. As it happens, this is exactly the form of dependence arbitrarily assumed by Sutcliffe and Montmitonnet [12] in their model of foil rolling. The roll load per unit width is given by

$$W = \int_{x_a}^{x_d} p(x)dx \quad (14)$$

The roll torque per unit width is given by

$$M = \int_{x_a}^{x_d} xp(x)dx + 0.5(\sigma_1 t_1 - \sigma_2 t_2) \quad (15)$$

3. NUMERICAL IMPLEMENTATION

As numerical problems associated with solving the foil rolling problem present a significant barrier to effective implementation, in this section we present a step-by-step guide to the solution method used. We cast the equations in non-dimensional forms, with the independent variables normalised as below:

$$X = \frac{xE_R^*}{RY_e};\ T = \frac{tE_R^{*2}}{RY_e^2};\ U = \frac{\mu E_R^*}{Y_e};$$

$$P = \frac{p}{Y_e};\ \Sigma_1 = \frac{\sigma_1}{Y_e};\ \Sigma_2 = \frac{\sigma_2}{Y_e};$$

where Y_e is an effective yield stress, $Y_e = Y_S - 0.5(\sigma_1 + \sigma_2)$ and σ_1 and σ_2 are the entry and exit stresses, respectively. The rationale behind using this effective yield stress will become clear when considering the results, section 4. Other parameters are normalised as below:

$$\Sigma = \frac{\sigma}{Y_e};\ Q = \frac{qE_R^*}{Y_e^2};\ \hat{W} = \frac{WE_R^*}{RY_e^2};$$

$$\hat{M} = \frac{ME_R^{*2}}{RY_e^3};$$

The solution now proceeds as follows:

We first assume a circular roll arc. An estimate must be made of a roll arc length which will exceed the actual contact arc, to be used in subsequent roll elasticity calculations (item 8),

The entry point X_a is determined from the roll shape, the roll separation and the inlet thickness T_1. The program starts with an estimate for the neutral point X_n, based on a previous iteration where appropriate.

Integrate the pressure and tension stress variation through the bite, with the given deformed roll shape and neutral position, as described in steps 4 to 6 below. The integration starts with the inlet boundary conditions P = 0, $\Sigma = \Sigma_1$.

In the inlet elastic slip region simultaneously integrate the dimensionless forms of equations 6 and 7, using a standard variable-step size 2nd/3th Order Runge-Kutta scheme [12]

$$\frac{d\Sigma}{dX} = -(\Sigma + P)\frac{dT}{TdX} - \frac{2UP}{T} \quad (16)$$

$$\frac{dP}{dX} \cong -\frac{E_S^*}{Y_e}\frac{dT}{TdX} + \frac{\nu_S}{1-\nu_S}\frac{2UP}{T} \quad (17)$$

The end of the elastic slip region is reached when the yield criteria is satisfied, $P+\Sigma=Y_S/Y_e$ at $X = X_b$.

To continue the integration first assume that plastic slip is occurring, integrating the pressure using the dimensionless form of Eq.10:

$$\frac{dP}{dX} = \frac{Y_S}{Y_e}\frac{dT}{TdX} + \frac{2Q}{T} \quad (18)$$

where the slipping friction expression, Eq.8, becomes $Q = \pm UP$. As the integration proceeds, a test is carried out to determine whether sticking is occurring, in which case an alternative differential equation is used as described below. When $\frac{dT}{dX} < 0$, the strip is plastic. Now if the shear stress Q for sticking, $-\frac{C_1 E_S^*}{2Y_e}\frac{dT}{dX}$, (Eqn. 12), is less than that for slipping, UP, then this is a plastic sticking region and the following equation should be used:

$$\frac{dP}{dX} = -\frac{C_1 E_S^*}{Y_e}\frac{dT}{TdX} \quad (19)$$

When $\frac{dT}{dX} > 0$, then elastic unloading of the strip is occurring. Now, where $Q = -\frac{C_2 C_3 E_S^*}{2Y_e}\frac{dT}{dX} < UP$ is satisfied (using Eqn. 14), this is an elastic sticking region and the following equation should be used:

$$\frac{dP}{dX} = -\frac{C_2 E_S^*}{Y_e}\frac{dT}{TdX} \quad (20)$$

Since the coefficient C_2 in Eq.20 is close to C_1 in Eq.19 and the pressure gradient is small in the elastic sticking region, Eq.20 for the plastic sticking region is used throughout the sticking region for simplicity without significant change in the solution. The end of the exit plastic slip region X_c is taken as the position where the roll slope is zero, with $\frac{dT}{dX} = 0$.

In the exit elastic slip region equations 17 and 18, suitably modified to account for the reversed direction of slip, are again integrated from X_c. The exit position X_d is determined when the normal pressure P falls to zero. A measure of the error in the estimate of the neutral point position ε_S is given by the difference in the calculated exit tension stress Σ_d and the required value Σ_2, i.e. $\varepsilon_S = \Sigma_d - \Sigma_2$.

The neutral position is found using a standard solver which uses a combination of bisection, secant and inverse quadratic interpolation [13] to adjust the neutral position, running through steps 2 to 6 for each value of X_n, until $\varepsilon_s = 0$. This typically taking 10 iterations. At the end of this step we have solved the pressure distribution for a given roll shape. It now remains to find a deformed roll shape which is consistent with this pressure distribution.

Recall (item 1) that a roll arc has been identified for the elasticity a calculation which encompasses the contact arc. The roll elastic deformation B(X) is calculated at N nodes located at equal intervals C along this arc using the influence coefficients given by Fleck et al [5] for an elastic half-space

$$B(X_i) = \sum_{j=1}^{j=N} (D_{ij} - D_{1j}) P_j \tag{21}$$

where k=i-j.

The strip thickness T through the bite is then given by $T = T_0^{(n)} + 2B$ (22)

where T_0 is the undeformed roll gap shape. Interpolation between the roll shape at the node positions is performed during the numerical integration of the pressure distribution using linear interpolation. The roll shape is then updated using a relaxation factor *e*, according to the following expression

$$T^{(n+1)} = eT + (1-e)T^{(n)} \tag{23}$$

Typically, values of *e* in the range 0.025 to 0.20 are used, with the smaller values for the more severe roll deformation cases. In general the roll shape associated with the new solution $T^{(n+1)}$ will not have the required exit gauge. Hence, it is necessary to apply some rigid body displacement of the rolls. This is done by changing the deformed and undeformed roll gap shapes by a uniform amount

$$T^{(n+1)} = T^{(n+1)} - e\varepsilon_t \tag{24}$$

$$T_0^{(n+1)} = T_0^{(n+1)} - e\varepsilon_t \tag{25}$$

where $\varepsilon_t = T_d - T_1$ is the error in the exit gauge. The same relaxation factor *e* is used here as for the roll shape iteration, Eq.24.

The whole process, from (2) to (8), is then repeated until the roll shape converges, as estimated by the criterion

$$\max\left(abs(T - T^{(n)}\right) < \Delta \tag{26}$$

where a tolerance of Δ=0.01 T_1 is typical used. This normally takes between 100 to1000 iterations, with the more severe cases taking approximately three CPU hours on a Sun workstation.

The non-dimensional roll load per unit width is given by

$$\hat{W} = \int_{X_a}^{X_d} P(X)\,dX \tag{27}$$

4. Theoretical results

Results are presented in two sections. The results in section 4.1 confirm that the model agrees with that of Fleck et al [5]. In section 4.2 new results are presented for the forward slip and for the effects of end tensions.

4.1 Roll shape, contact pressure, load and torque.

For a strip inlet thickness of 0.20mm, the roll shape is approximately circular. When the inlet thickness is reduced to 0.10mm, the contact pressure goes up resulting in some roll deformation, while at an inlet gauge of 0.03mm there is a substantial sticking region in the middle of the bite which is nearly flat. These results reproduce the effects observed by Fleck et al [5] for thin foil rolling, including a flat central region, a nearly Hertizan pressure distribution and a pressure spike just after the neutral zone. Although elastic slip regions at entry and exit have been included in the analysis, these do not appear to play a significant role. Results are in good agreement with those of Fleck et al [5].

4.2 The effect of tensions on roll load and forward slip

The load is only slightly changed where unequal tensions are chosen. This demonstrates that the effect of equal tensions has been accommodated by normalising the relevant parameters using the effective yield stress, subtracting off the mean tension stress. Although this is a standard procedure in thick strip rolling, it is not obvious that the result will hold for thin strip rolling. However the result can be understood by examining those parts of the governing equations where the yield stress or effective yield stress occur, i.e. in equations 18 to 21. Due to the small slopes in thin strip conditions, the relevant terms in these equations are negligible.

5. CONCLUSIONS

A new model is presented for cold rolling of thin strip and foil. In general the model adopts the modelling approach of Fleck et al [5]. However instead of assuming a flat central region and solving the pressure distribution in the central neutral zone by inverting the elasticity solution for this region, small elastic and plastic strains in this central region are considered. This leads to an explicit solution for the pressure distribution in the central region, resulting in a much simpler, faster and more robust numerical algorithm. These advantages will facilitate the incorporation of additional complications such as a more sophisticated friction model. The load and torque predicted by this model are in good agreement with those of Fleck et al [5]. Parametric studies show that the effect of equal tensions is equivalent to reducing the yield stress by the value of the tension stress. The model predicts an increase in forward slip with increasing exit tension or decreasing entry tension, an effect well known from industrial practice.

Acknowledgement

The authors express their sincere thanks to Maa Yashodhara Industrial Training Institute, Sambhuganj, Banka, Bihar, India and colleagues of that stream. The corresponding author expresses his earnest thanks to the co-authors Miss Jyoti Raman and Miss Priya, Maa Yashodhara Industrial Training Institute, Sambhuganj, Banka, Bihar, India. The author also acknowledges to reviewers of JME for their careful reading of our manuscript.

References

1. *von Karmann T (1925), "Beitrag zur theorie des Walzvorganges. Z. angeur. Math. Mech., Vol. 5, 139-144.*
2. *Jortner D, Osterle J F and Zorowski C F (2011), "An analysis of cold rolling", Int. J. Mech. Sci., Vol. 2(1), 179-194.*
3. *Johnson K L and Bentall R H (2009), "The onset of yield in the cold rolling of thin strip", J. Mech. Phys. Solids, Vol. 17(1), 253 -261.*
4. *Fleck N A and Johnson K L (2017), "Towards a new theory of cold rolling thin foil", Int. J. Mech. Sci., Vol. 29(7), 507-524.*
5. *Fleck N A, Johnson K L, Mear M E and Zhang L C, (2012). Cold rolling of foil, Proc. Instn. Mech. Engrs, 1992, vol. 206, 119-131.*
6. *Sutcliffe M P F and Rayner P J, (2018), Experimental measurements of load and strip profile in thin strip rolling, Int. J. Mech. Sci., Vol. 40, 887-889.*
7. *Yuen W Y D, Dixon A and Nguyen D N (2016), "The Modelling of the Mechanics of the Deformation in Flat Rolling", J. Mech. Proc. Tech., Vol. 60(1), 87-94.*
8. *Domanti S A, Edwards W J and Thomas P J (2014), A model for rolling and thin strip rolling, AISE Annual Convention, Cleveland, Ohio, USA.*
9. *Domanti S A, Edwards W J, Thomas, P. J. and Chefneux, I. L. (2014), Application of foil rolling models to thin steel strip and temper rolling, 6th International Rolling Conference, Dussedorf, 422-429.*
10. *Gratacos, P., Montmittonet, P., Fromholz, P. and Chenot, J. L. (2012), A plane-strain elastic finite-element model for cold rolling of thin strip, International Journal of Mechanical Sciences, Vol. 34(3), 195-210.*
11. *Johnson, K. L. (1985), Contact Mechanics, Cambridge University Press.*
12. *Sutcliffe, MPF. And Montmittonet, P. (1999), A coupled tribological and mechanical model for thin foil rolling in the mixed lubrication regime, To be presented at the Institute of Materials Conference on Metal Rolling Processes, London.*

