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REAL CODED VARIABLE POPULATION SIZE GENETIC ALGORITHM FOR FIXED CHARGE TRANSPORTATION PROBLEM IN MULTI-STAGE SUPPLY CHAIN NETWORK

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ABSTRACT

In this paper a mathematical model is developed for Multi-Stage Supply Chain Network (MSCN) associated with fixed charge for each route and proposed a solution procedure based on real coded variable population size Genetic Algorithm (RC-VPGA). The supply chain is often represented as a network called a supply chain network (SCN) which comprised of nodes that represent facilities (suppliers, plants, distribution centers and customers). These nodes are connected by arcs that represent the production flow. The objective of this paper is to select the optimum set of suppliers, plants, distribution centers (DCs) to be opened and the distribution network design to satisfy the customer demand with minimum total distribution cost. In many distribution problems the transportation cost consists of fixed charges, which are independent of the quantities transported, and variable costs, which are proportional to the quantities transported. The problem chosen goes beyond the traditional mathematical programming and it becomes Non- Polynomial (NP) hard while considering the fixed charges. Our new idea lies on the adoption of fixed charges for each production flow between the stages. The performance of the proposed methodology is compared with approximate and lower bound solutions. The comparison reveals that the RC-VPGA generates better solution than an approximation method and is capable of providing solution closer to the lower bound solution of the problem.

Keywords: *Real Coded Variable Population Size Genetic Algorithm, Multi-Stage, Supply Chain Network, Fixed Charge Transportation Problem*

1. Introduction

Supply chain activities start with a customer order and end if a satisfied customer has paid for the purchase. In practical applications, a plant may receive material from several suppliers and then supply the products to customers through several DCs. There are usually several stages, including suppliers, plants, DCs, and customers in designing MSCN .The supply chain consists of all stages involved, directly or indirectly, in fulfilling a customer request. Managing MSCN has been a popular area of research (Syarif et al., 2002, Chopra and Meindl, 2003; Yeh, 2005, 2006; Hang et al., 2006; Jawahar and Balaji, 2009) and has received significant attention because MSCN can achieve great success in satisfying customer demands in the best possible way. The appropriate MSCN design depends on both the customer's needs and the roles of the stages involved in satisfying those needs.

In many MSCN problems, the transportation cost consists of a fixed charge, independent of the amount transported along with a variable cost that is proportional to the amount shipped.

The fixed charge may represent toll charges on a highway; landing fees at an airport; set-up costs in production systems or cost of building roads in transportation systems, etc (Palekar et al., 1990). The objective of every MSCN problem is to find a network strategy that can provide the least cost for the physical distribution flow. Fixed charge transportation problem (FCTP) is much more difficult to solve due to the presence of fixed costs which causes discontinuities in the objective function and renders it unsolvable by the direct application of the transportation algorithm (Kim and Pardalos, 1999, Gen et al., 2005). FCTP is often formulated and solved as a mixed integer programming problem. However, these methods are generally inefficient and computationally expensive because they do not take advantage of the special network structure of the FCTP (Steinberg, 1970). As a result, much research effort has been devoted to device heuristic solution procedures which, run in reasonable computer time and yield solutions of acceptable quality (Jo et al., 2007).

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Syarif et al., (2002) implemented Genetic Algorithm (GA) using a Prüfer number representation for coding the spanning tree structure (st-GA) to solve the MSCN problem with an aim to satisfy the customer demand with minimum cost. Adlakha and Kowalski (2003) presented a simple heuristic algorithm for the solution of small fixed-charge problems. In addition that their proposed method is more time consuming than the algorithms to solve a regular transportation problem, and it provides a good foundation to solve small problems by hand. Shi et al., (2005) proposed a Variable Population-size Genetic Algorithm (VPGA) to determine the population size and the alternation between generations. Yeh (2005) developed a hybrid heuristic approach for the problem considered by Syarif et al., (2002). The approach is a combination of a greedy method, linear programming technique and three local search methods. Yeh (2006) also proposed a memetic algorithm which is a combination of GA, greedy heuristic, and local search methods and extensively investigated on the randomly generated problems. Hang et al., (2006) introduced an improved genetic algorithm based on the Prüfer number (IPE-GA) and the effective capacity coding to construct the chromosome to search the whole solution space of the unbalanced multi stage logistics system. Jo et al., (2007) proposed the spanning tree-based GA approach to solve non-linear FCTP. The main disadvantages of using the Prüfer number are that the logistic patterns decoded from many Prüfer numbers are not feasible solutions. Also, in an unbalanced logistics system, some logistics patterns are impossible no matter what Prüfer number is decoded. Adlakha et al., (2007) developed a simple heuristic to find more for less solution for both classical and fixed cost transportation problems. Besides, they stated that the existing analytical algorithms such as branch and bound for solving FCTP are useful only for small problems. Very recently Jawahar and Balaji (2009) used GA for the two stage supply chain distribution problem associated with a fixed charge with the assumption that the capacity of the DC is very large.

In real practice, the fixed charges are incurred for operating the supplier to transport the raw materials to plant. This is not considered by the researchers in literature. Apart from that, the fixed charges for each supplier to plant, each plant to distribution center and each distribution center to customer are not considered while solving MSCN in the previous works. This will leads to wrong interpretation in calculating the total transportation charges for multi stage logistics environment. This recent attention accorded to intelligent heuristic search procedures provides the methodological motivation for our use of genetic search procedures. As a result, current research examines the use of algorithmic solution procedures. On class of solution procedures that are receiving renewed attention, and considered in this work is real coded variable population size Genetic Algorithm (RC-VPGA).

We consider a specific sub-group of GA. Conceptually, any instance of a supply chain configuration is represented computationally by a 'population'. We propose a special type of GA that introduces the probability of 'death' (or exclusion) for individual members of a population describing an instance of a supply chain configuration and it is known as the Variable Population-size Genetic Algorithm (VPGA). The VPGA considers that the better individuals are put into the next population through comparing individuals in the previous and new reproductive populations. The main idea of the VPGA is that parents are neither dead after their reproduction right away, nor living forever. In fact, the individual is apt to die when it is old. During the death process, each individual's survival is determined according to its unique probability. In general, the 'disease' or the 'war' among the individuals will reduce the size of the population sharply. In the VPGA, the size of the population is reduced to an initial size when it reaches some given limitation, of which the individuals with higher fitness have more opportunities to survive and this process is called "war/disease process". The maximum lifetime depends on the fitness of the corresponding individual, while the age is incremented at each generation by one. Individuals are removed from the population when their ages reach the value of their predefined maximal lifetime.

The paper is organized as follows. Section 2 contains the assumptions and mathematical model for the MSCN problem. In section 3, the RC-VPGA method is discussed in detail. Section 4 illustrates how the proposed methodology is used to solve the MSCN problem through a numerical example. Section 5 details about lower bound value and approximate solution. Concluding remarks and future works are given in section 6. References are given in section 7 and nomenclature is provided in section 8.

2. Problem Description

The cost of supply chain system observes nearly 30% to 35% of the cost of the product. Globalization forces the customer to fix the cost of the product. Suppliers are mortal need of novel approach for transporting the products with minimum cost and earn considerable profit. In this scenario, the selection and allocation of quantities from supplier to plant, plant to DC and DC to the customer, plays an important role in controlling the cost of the product.

In practical situations, the fixed charges are incurred for operating the supplier to transport the raw materials to plant and incurred between each route. Considering the real situation existing in industrial firms, this paper proposes a mathematical model to describe the MSCN network problem involving fixed charge for each route. This is the novelty of our mathematical model.

2.1 Assumptions

The MSCN satisfies the following assumptions:

- The demand of each customer must be satisfied
- The number and capacities (or demands) of suppliers, plants, DCs and customers are known in advance.
- The unit transportation cost between each echelons and the fixed charge for operating each route between echelons are well known in advance.
- The number of open facilities is limited.

2.2 Problem environment

There are '*S*' suppliers with the capacity sc_i (i varies from 1 to ns) supplying the raw materials to '*P*' plants with the capacity pc_j (j varies from 1 to np) and distributing the products to '*C*' number of customers having demands cd_l (l varies from 1 to nc) through '*D*' number of distribution centers with the capacity dc_k (k varies from 1 to nd). The fixed charge incurred for each supplier i to plant j (f_{ij}), each plant j to DC k (f_{jk}) and each DC k to customer l (f_{kl}) are considered for finding the distribution cost.

2.3 Objective function and mathematical model

Minimizing cost as objective function for the fixed charge problem under consideration. The objective is to identify supplier, plant and DC to be opened and determine the quantities to be transported between the facilities with minimum total distribution cost (total fixed charges and transportation costs). It will be sum of the costs to transport the raw materials from suppliers to plants, distribute the products from plants to DCs, and distribute the products from DCs to the customers and the fixed charges incurred between each supplier to plant, each plant to DC and each DC to customer.

The multi stage FCTP is formulated in using the mixed 0-1 integer programming model as follows:

$$\begin{aligned} \text{Minimize } z = & \sum_{i=1}^{ns}\sum_{j=1}^{np} c_{ij}x_{ij} + \sum_{j=1}^{np}\sum_{k=1}^{nd} c_{jk}x_{jk} + \\ & \sum_{k=1}^{nd}\sum_{l=1}^{nc} c_{kl}x_{kl} + \sum_{i=1}^{ns}\sum_{j=1}^{np} f_{ij}d_{ij} + \\ & \sum_{j=1}^{np}\sum_{k=1}^{nd} f_{jk}d_{jk} + \sum_{k=1}^{nd}\sum_{l=1}^{nc} f_{kl}d_{kl} \end{aligned} \tag{1}$$

$$\sum_{j=1}^{np} x_{ij} \leq sc_i \tag{2}$$

$$\sum_{k=1}^{nd} x_{jk} \leq pc_j \tag{3}$$

$$\sum_{l=1}^{nc} x_{kl} \leq dc_k \tag{4}$$

$$\sum_{i=1}^{ns}\sum_{j=1}^{np} x_{ij} \leq \sum_{i=1}^{ns} sc_i \tag{5}$$

$$\sum_{k=1}^{nd}\sum_{j=1}^{np} x_{jk} \leq \sum_{j=1}^{np} pc_j \tag{6}$$

$$\sum_{k=1}^{nd}\sum_{l=1}^{nc} x_{kl} \leq \sum_{k=1}^{nd} dc_k \tag{7}$$

$$\sum_{k=1}^{nd}\sum_{l=1}^{nc} x_{kl} = \sum_{l=1}^{nc} cd_l \tag{8}$$

$$\sum_{i=1}^{ns} sc_i, \sum_{j=1}^{np} pc_j, \sum_{k=1}^{nd} dc_k \geq \sum_{l=1}^{nc} cd_l \tag{9}$$

$d_{ij} = 1$ if transportation takes place from *supplier* to plant
$\quad = 0$ otherwise

$d_{kl} = 1$ if transportation takes place from plant to DC
$\quad = 0$ otherwise

$d_{ij} = 1$ if transportation takes place from DC to customer
$\quad = 0$ otherwise

$x_{ij}, x_{jk}, x_{kl} \geq 0$ for all i, j, k and l

The objective function in eq. (1) minimizes the sum of transportation cost and fixed charges incurred for operating each route between the facilities.

Equations (2) – (4) are the network balance constraints that ensure facility capacities at all suppliers, plants and distribution centers are not violated. Eq. (5) indicates that the sum of quantity transferred from all the suppliers to all the plants must be less than or equal to the sum of suppliers capacity. Eq. (6) states that the sum of quantity transferred from all the plants to all the distribution centers must be less than or equal to the sum of plants capacity. Eq. (7) implies that the sum of quantity transferred from all the distribution centers to all the customers must be less than or equal to the sum of distribution centers capacity. Eq. (8) conditions that the sum of quantity transferred from, all the distribution centers to all the customers must be equal to the sum of customers demand. Eq. (9) ensures that the total flows of products through all echelons meet total consumer demand.

3. Proposed Methodology

The steps involved in the proposed methodology are explained with the help of flow chart Fig. 1 and Fig 2.

Step 1: Chromosome representation and population Initialization.
Step 2: Evaluation of chromosomes
Step 3: Application of VPGA Operators
Step 3.1: Selection of chromosomes for reproduction
Step 3.1.1: Calculation of new_fitness value
Step 3.1.2: Calculation of total new_fitness values
Step 3.1.3: Probability of selection of chromosomes
Step 3.1.4: Computation of cumulative probability
Step 3.1.5: Selection of new chromosomes
Step 3.2: Crossover module
Step 3.3: Mutation module
Step 3.4 Replacement strategy
Step 3.4.1: Evaluation of chromosomes after mutation
Step 3.4.2: Checking the survival of the chromosomes
Step 3.4.3: "War/disease" process.
Step 4: Termination module

Above steps are explained in the following section with the numerical example.

4. Implementation of RC-VPGA with Numerical Illustration

To demonstrate the proposed methodology in multi stage supply chain network, the following illustrated problem is generated randomly and solved. To validate the efficiency of the proposed methodology the results are compared with lower bound value and an approximate solution. Table 1 shows capacity of supplier, plant, DC, demand of the customer, unit transportation cost between facilities and fixed charges incurred for each route.

Table 1: Capacity of Supplier, Plant, DC, Demand of the Customer, Unit Transportation Cost between Facilities & Fixed Charges Incurred for each Route

	P_1	**P_2**	**P_3**	**P_4**	**P_5**	**SC**
S_1	C_{ij} 2	4	7	5	6	260
	f_{ij} 528	152	504	611	75	
S_2	5	3	2	5	3	300
	733	86	738	87	12	
S_3	6	5	4	4	2	450
	28	639	70	111	553	
S_4	4	7	8	6	9	360
	727	741	462	77	788	
S_5	3	4	5	6	5	480
	135	119	758	436	37	
PC	350	450	600	550	450	1850
						2400
	DC_1	**DC_2**	**DC_3**	**DC_4**	**DC_5**	**PC**
P_1	C_{jk} 9	4	6	5	4	350
	f_{jk} 855	679	296	474	909	
P_2	3	3	5	7	6	450
	907	88	318	328	562	
P_3	2	7	9	4	6	600
	295	373	581	36	407	
P_4	6	5	7	6	4	550
	379	621	878	206	74	
P_5	4	3	4	2	5	450
	20	733	312	801	443	
DCC	500	550	400	600	450	2400
						2500
	C_1	**C_2**	**C_3**	**C_4**	**C_5**	**DCC**
DC_1	C_{kl} 5	2	7	3	4	500
	f_{kl} 443	12	841	349	41	
DC_2	3	6	2	7	7	550
	942	696	967	127	306	
DC_3	2	8	7	2	1	400
	274	63	353	369	464	
DC_4	4	7	8	6	9	600
	547	668	339	247	10	
DC_5	3	5	3	4	6	450
	232	24	505	23	632	
CD	180	280	160	340	240	2500
						1200

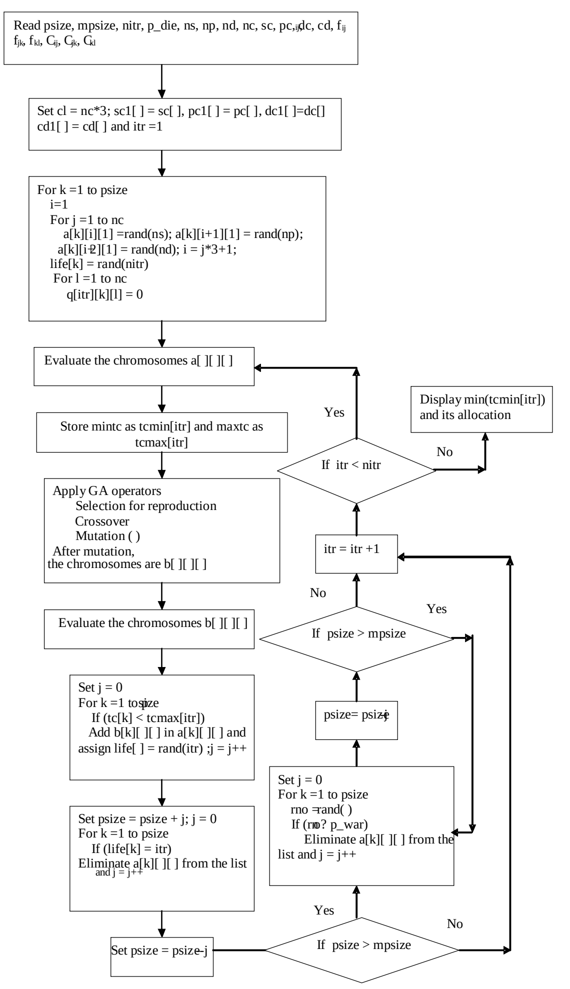

Fig. 1 Proposed RC-VPGA Methodology

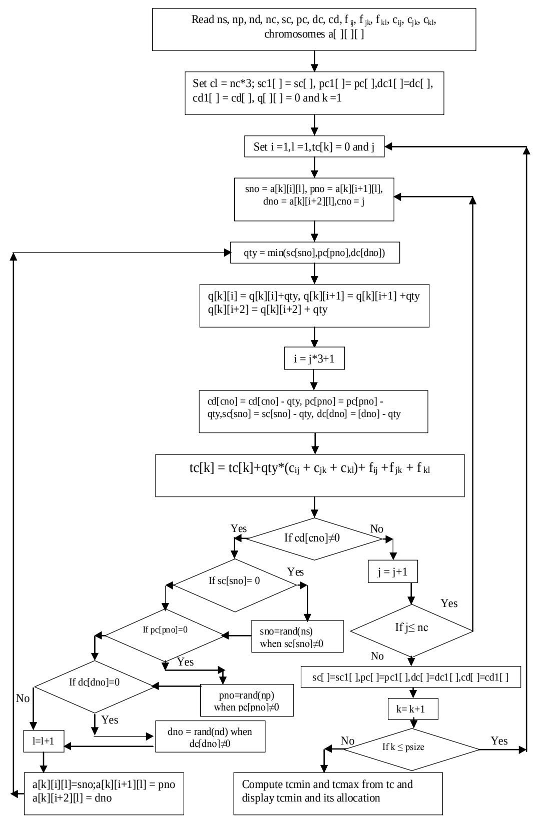

Fig. 2 Evaluation of Chromosomes

Step 1: Chromosome representation and population initialization

The length of the sub-string of each chromosome is equal to number of stages and the total length of the string (*Chr_len*) is equal to the product of number of customers and number of stages. The first three digits (1 4 4) represents the supplier, the plant and the distribution center number that are generated randomly for the zeroth customer.

Step 2: Evaluation of chromosomes

Fitness value (total distribution cost) of all chromosomes are evaluated and presented with their life in Table 2. The minimum and maximum of the fitness value is stored as mintc and maxtc for further genetic processes.

Step 3: Application of VPGA Operators

3.1: Selection of chromosomes for reproduction

Table 2: Chromosomes, Life and Total Distribution Cost of Initial Population

Chr. No.	Chromosome	Life	fit(c)
1	1 4 4 3 2 3 2 0 2 1 4 3 2 1 2	4	23828
2	1 3 3 1 2 1 3 1 2 2 2 0 4 1 1	4	24470
3	3 2 1 2 1 3 3 0 0 2 2 2 1 4 3	5	28701
4	2 1 4 2 2 1 4 4 2 0 3 2 1 0 4	4	27568
5	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	4	29059
6	3 4 1 4 1 3 4 1 1 3 2 3 0 1 0	5	26425
7	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	4	25620
8	4 2 3 3 4 1 3 1 0 0 0 1 1 0 2	3	26831
9	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	1	27892
10	0 1 4 3 4 0 0 1 3 2 0 1 3 3 0	4	24712

Few chromosomes are selected from the initial population as parents intended for crossover and produce off spring. Roulette wheel method is used in this work for selecting the chromosomes for reproduction.

Step 3.1.1: The new fitness value is calculated using the exponential scaling function.

$$new\_fit\left(C_i\right) = e^{-t.\,fit\left(C_i\right)} \quad (10)$$

In this work, the value of t is fixed as 0.00005 based on trials, to obtain superior solutions.

Step 3.1.2: The total new_fitness values of the population is calculated based on the expression

$$F = \sum_{i=1}^{pop\_size} new\_fit(C_i) \quad (11)$$

Step 3.1.3: The probability of a selection p_i for C_i (i = 1… *pop_size*) is calculated using the equation

$$p_i = \frac{new\_fit\left(C_i\right)}{F} \quad (12)$$

Step 3.1.4: Cumulative probability $cp\left(C_i\right)$ for i = 1… *pop_size* is computed using

$$cp(C_i) = \sum_{j=1}^{i} p_j \quad (13)$$

Step 3.1.5: A random number is generated between 0 and 1 for each chromosome. A chromosome (C_i) is selected for which the random number satisfies the following constraint.

$$cp(C_{i-1}) < r \le cp(C_i) \quad (14)$$

Table 3 represents the selected new chromosomes for reproduction as per the above steps.

Table 3: Parameters for New Chromosomes

Chr No	new_ fit	Prob.	Cum Prob	Ran no.	Sel chr	New chr
1	0.5512	0.1068	0.1068	0.3140	4	1'
2	0.5424	0.1052	0.2120	0.8620	9	2'
3	0.4880	0.0946	0.3066	0.9470	10	3'
4	0.5020	0.0973	0.4039	0.6810	7	4'
5	0.4836	0.0937	0.4976	0.4750	5	5'
6	0.5165	0.1001	0.5977	0.6790	7	6'
7	0.5270	0.1022	0.6999	0.4760	5	7'
8	0.5113	0.0991	0.7990	0.0010	1	8'
9	0.4979	0.9650	0.8955	0.5850	6	9'
10	0.5391	0.1045	1.0000	0.3900	4	10'

Step 3.2: Crossover module

In this work, single point crossover is adopted. By trial, the probability of crossover (p_c) is fixed as 0.6.

Step 3.2.1: A random number (r) is generated for each new chromosome between 0 and 1. If the random number is less than or equal to probability of crossover then the chromosome is selected for crossover. Chromosomes 3', 4', 5' and 7' are selected. Table 4 represents the selected chromosomes for crossover.

Table 4: Chromosome Selection Process for Crossover

Ch. No	Chromosome	Ran. no	S/ NS	New ch.
1'	2 1 4 2 2 1 4 4 2 0 3 2 1 0 4	0.967	NS	1'
2'	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	0.701	NS	2'
3'	0 1 4 3 4 0 0 1 3 2 0 1 3 3 0	0.402	S	3"
4'	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	0.239	S	4"
5'	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	0.521	S	5"
6'	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	0.750	NS	6'
7'	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	0.345	S	7"
8'	1 4 4 3 2 3 2 0 2 1 4 3 2 1 2	0.631	NS	8'
9'	3 4 1 4 1 3 4 1 1 3 2 3 0 1 0	0.712	NS	9'
10'	2 1 4 2 2 1 4 4 2 0 3 2 1 0 4	0.805	NS	10'

Step 3.2.2: A random number (cutting point) having value less than the chromosome length is generated for each set of selected chromosome for crossover. For example, two random numbers are generated when four chromosomes are selected for crossover. The chromosomes after crossover are presented in Table 5.

Table 5: Chromosomes after Crossover

Ch.No	After crossover -Child
1'	214 221 442 032 104
2'	104 424 332 000 413
3"	014 3***14 434 204 432***
4"	***400 132 013 30***2 201
5''	310 132 021 302 ***310***
6'	144 342 044 322 201
7"	***201*** 132 021 302 201
8'	144 323 202 143 212
9'	341 413 411 323 010
10'	214 221 442 032 104

*Crossed are shown by bold and italic fonts

Step 3.3: Mutation module

Random numbers are generated for each gene in the chromosomes after crossover. In this work, the probability of mutation p_m is fixed as 0.045 based on trials.

Step 3.4: Replacement strategy

Step 3.4.1: Chromosomes after mutation are evaluated as per step 2 and shown in Table 6. Based on the conditions below, (previous min tc and max tc), the chromosomes after mutation are selected and listed in Table 6.

$$fit(C_i) \leq \max tc \text{ and } fit(C_i) \leq \min tc \qquad (15)$$

Select the chromosome, otherwise not select

Table 6: Evaluated Chromosomes after Mutation

Ch. No	Chromosome	*fit(C)*	Life	S/NS
1	2 1 4 2 2 4 1 4 2 0 3 2 1 0 1	27233	4	S
2	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	27892	4	S
3	0 1 4 3 1 4 4 3 4 2 0 2 2 3 1	30041	5	S
4	1 0 0 1 3 2 0 1 3 3 0 2 2 3 4	26903	4	S
5	1 1 0 1 3 2 0 2 3 2 0 2 3 1 0	26392	4	S
6	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	29155	5	S
7	2 0 1 1 3 2 0 2 1 3 0 2 2 1 1	31341	4	S
8	2 4 4 3 2 3 2 0 2 1 4 3 2 1 1	26841	3	S
9	2 4 1 4 1 3 1 4 1 2 3 3 0 1 0	27128	1	S
10	4 1 4 2 1 2 4 4 2 0 3 0 1 0 1	27817	4	S

Step 3.4.2: The life of all chromosomes is compared with the current iteration number to check the survival of the chromosomes for the next generation. Table 7 represents all chromosomes including initial population and the selected chromosomes after mutation and their life.

Table 7: Survival Check for Chromosomes

Ch. No	Chromosomes	Life	Survival	S/NS
1	2 1 4 2 2 4 1 4 2 0 3 2 1 0 1	1	Yes	S
2	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	3	Yes	S
3	1 0 0 1 3 2 0 1 3 3 0 2 2 3 4	1	Yes	S
4	1 1 0 1 3 2 0 2 3 2 0 2 3 1 0	4	Yes	S
5	2 4 4 3 2 3 2 0 2 1 4 3 2 1 1	4	Yes	S
6	2 4 1 4 1 3 1 4 1 2 3 3 0 1 0	4	Yes	S
7	4 1 4 2 1 2 4 4 2 0 3 0 1 0 1	1	Yes	S
8	1 4 4 3 2 3 2 0 2 1 4 3 2 1 2	4	Yes	S
9	1 3 3 1 2 1 3 1 2 2 2 0 4 1 1	4	Yes	S
10	3 2 1 2 1 3 3 0 0 2 2 2 1 4 3	5	Yes	S
11	2 1 4 2 2 1 4 4 2 0 3 2 1 0 4	4	Yes	S
12	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	4	Yes	S
13	3 4 1 4 1 3 4 1 1 3 2 3 0 1 0	5	Yes	S
14	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	4	Yes	S
15	4 2 3 3 4 1 3 1 0 0 0 1 1 0 2	3	Yes	S
16	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	1	Yes	S
17	0 1 4 3 4 0 0 1 3 2 0 1 3 3 0	4	Yes	S

Step 3.4.3: Based on trails, the maximum population size is fixed as 1.5 times of the initial population. The actual chromosomes available after life survival check are 17. This is more than that of the maximum population size. Hence, the process of "war/disease" is adopted to reduce the population size. A probability of war/disease is fixed as 0.4 based on trials. A random number is generated for each chromosome between 0 and 1 and selected chromosomes are tabulated in Table 8.

Table 8: Total Chromosomes before War

Chr. No	Chromosomes	Life	Ran. no.	S/NS
1	2 1 4 2 2 4 1 4 2 0 3 2 1 0 1	1	0.0110	NS
2	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	3	0.6730	S
3	1 0 0 1 3 2 0 1 3 3 0 2 2 3 4	1	0.8710	S
4	1 1 0 1 3 2 0 2 3 2 0 2 3 1 0	4	0.8130	S
5	2 4 4 3 2 3 2 0 2 1 4 3 2 1 1	4	0.8190	S
6	2 4 1 4 1 3 1 4 1 2 3 3 0 1 0	4	0.7090	S
7	4 1 4 2 1 2 4 4 2 0 3 0 1 0 1	1	0.5180	S
8	1 4 4 3 2 3 2 0 2 1 4 3 2 1 2	4	0.1220	NS
9	1 3 3 1 2 1 3 1 2 2 2 0 4 1 1	4	0.6610	S
10	3 2 1 2 1 3 3 0 0 2 2 2 1 4 3	5	0.4830	S
11	2 1 4 2 2 1 4 4 2 0 3 2 1 0 4	4	0.0150	NS
12	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	4	0.5630	S
13	3 4 1 4 1 3 4 1 1 3 2 3 0 1 0	5	0.5960	S
14	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	4	0.8880	S
15	4 2 3 3 4 1 3 1 0 0 0 1 1 0 2	3	0.4810	S
16	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	1	0.8430	S
17	0 1 4 3 4 0 0 1 3 2 0 1 3 3 0	4	0.7500	S

The selected chromosomes for the next iteration with their life are listed in Table 9.

Table 9: Chromosomes after War for Next Iteration

Chr.No	Chromosomes	Life
1	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	3
2	1 0 0 1 3 2 0 1 3 3 0 2 2 3 4	1
3	1 1 0 1 3 2 0 2 3 2 0 2 3 1 0	4
4	2 4 4 3 2 3 2 0 2 1 4 3 2 1 1	4
5	2 4 1 4 1 3 1 4 1 2 3 3 0 1 0	4
6	4 1 4 2 1 2 4 4 2 0 3 0 1 0 1	1
7	1 3 3 1 2 1 3 1 2 2 2 0 4 1 1	4
8	3 2 1 2 1 3 3 0 0 2 2 2 1 4 3	5
9	3 1 0 1 3 2 0 2 1 3 0 2 2 0 1	4
10	3 4 1 4 1 3 4 1 1 3 2 3 0 1 0	5
11	1 4 4 3 4 2 0 4 4 3 2 2 2 0 1	4
12	4 2 3 3 4 1 3 1 0 0 0 1 1 0 2	3
13	1 0 4 4 2 4 3 3 2 0 0 0 4 1 3	1
14	0 1 4 3 4 0 0 1 3 2 0 1 3 3 0	4

Step 4: Termination module

The process of evaluation, selection, crossover, mutation and replacement strategy forms one generation/iteration in the execution of RC-VPGA. The algorithm terminates when it reaches the termination criteria and a best solution is obtained. The best solution is the optimal distribution plan that minimizes the total distribution cost. The best value after one thousand iterations is 16678. The optimum distribution plan is presented in Table 10.

Table 10: Optimal Distribution Plan

	P_1	P_2	P_3	P_4	P_5
S_1		160			
S_2		60	130		110
S_3			110		340
S_4				180	
S_5			110		

	DC_1	DC_2	DC_3	DC_4	DC_5
P_1					
P_2		160	60		
P_3	350				
P_4					180
P_5	110		340		

	C_1	C_2	C_3	C_4	C_5
DC_1		280			180
DC_2			160		
DC_3				340	60
DC_4					
DC_5	180				

5. Calculation of Lower Bound Value and Approximate Solution

To validate the efficiency and effectiveness of the proposed methodology, the results are compared with lower bound value and approximate solution.
The lower bound value is calculated as given below.

- Lower bound value z_1 can be obtained by relaxing the integer restrictions (Balinski, 1961) of the mathematical model discussed in section 2.3.

The objective function is

$$\text{Minimize } Z_1 = \sum_{i=1}^{ns} \sum_{j=1}^{np} C_{ij} X_{ij} + \sum_{j=1}^{np} \sum_{k=1}^{nd} C_{jk} X_{jk} + \sum_{k=1}^{nd} \sum_{l=1}^{nc} C_{kl} X_{kl} \qquad (16)$$

Where equivalent transportation cost

$$\left.\begin{aligned} C_{ij} &= c_{ij} + \frac{f_{ij}}{\min(sc_i, pc_j)} \\ C_{jk} &= c_{jk} + \frac{f_{jk}}{\min(pc_j, dcc_k)} \\ C_{kl} &= c_{kl} + \frac{f_{kl}}{\min(dcc_{ki}, cd_l)} \end{aligned}\right\} \qquad (17)$$

X_{ij}, X_{jk} and X_{kl} – Optimal quantities flowing from supplier i to plant j, plant j to DC k and from DC k to customer l respectively for the equivalent transportation cost.

Using the equation 17 equivalent transportation cost for various routes are calculated and tabulated in Table 11.

Table 11: Equivalent Transportation Cost for Various Routes

	P_1	P_2	P_3	P_4	P_5
S_1	4.031	4.585	8.938	7.350	6.288
S_2	7.443	3.289	4.460	5.290	3.040
S_3	6.080	6.420	4.156	4.247	3.223
S_4	6.077	9.058	9.283	6.213	11.189
S_5	3.386	4.264	6.579	6.908	5.082

	DC_1	DC_2	DC_3	DC_4	DC_5
P_1	11.442	5.940	6.846	6.354	6.597
P_2	5.016	3.196	5.795	7.729	7.249
P_3	2.590	7.678	9.022	4.060	6.904
P_4	6.758	6.129	9.195	6.375	4.164
P_5	4.044	4.629	4.780	3.780	5.984

	C_1	C_2	C_3	C_4	C_5
DC_1	7.461	2.043	12.256	4.026	4.171
DC_2	8.233	8.486	8.044	7.374	8.275
DC_3	3.522	8.225	9.206	3.085	2.933
DC_4	7.039	9.386	10.119	6.726	9.042
DC_5	4.289	5.086	6.156	4.068	8.633

Optimal quantities X_{Ij} , $X_{jk,}$ and X_{kl} are obtained by using LINGO package and shown in Table 12.

Table 12: Optimal Quantities X_{Ij} , $X_{jk,}$ and X_{kl}

	P_1	P_2	P_3	P_4	P_5
S_1					
S_2			100	200	
S_3		240	210		
S_4				50	
S_5		400			

	DC_1	DC_2	DC_3	DC_4	DC_5
P_1					
P_2		400			
P_3	240				
P_4					360
P_5	10			100	90

	C_1	C_2	C_3	C_4	C_5
DC_1		250			
DC_2			160		240
DC_3					
DC_4				100	
DC_5	180	30		240	

- Approximate solution z_2 is obtained by substituting the optimal quantities (X_{ij}, X_{jk} and X_{kl}) found from the objective function z_1 in the following equation

$$Minimize\ z_2 = \sum_{i=1}^{ns}\sum_{j=1}^{np} c_{ij}X_{ij} + \sum_{i=1}^{ns}\sum_{j=1}^{np} f_{ij}d_{ij} + \sum_{j=1}^{np}\sum_{k=1}^{nd} c_{jk}X_{jk} + \sum_{j=1}^{np}\sum_{k=1}^{nd} f_{jk}d_{jk} + \sum_{k=1}^{nd}\sum_{l=1}^{nc} c_{kl}X_{kl} + \sum_{k=1}^{nd}\sum_{l=1}^{nc} f_{kl}d_{kl} \tag{18}$$

Lower bound value and approximate solution are obtained by substituting optimal quantities in the equation 16 and 18 and shown in Table 12.

Lower bound value=15651

Approximate solution = 17368

The result obtained by proposed methodology 16678 is lesser than approximate solution and closer to lower bound value.

5. Results and Findings

Another four problems are generated randomly and solved by the proposed methodology. The results are compared with the lower bound value and approximate solution as given in Table 13.

Table 13: Comparison of Results

A	B	C	D	E	F	G
1	5*5*5*5	16678	15651	17368	6.56	-3.97
2	5*2*5*6	32663	30327	33696	7.70	-3.07
3	3*2*2*3	348000	339144	348000	2.61	0
4	4*2*2*4	276887	276218	276887	0.02	0
5	2*2*2*3	95200	95200	95200	0	0

A-S.No.; B-Problem size; C-Total distribution cost (z); D-LBV (z1); E- AS (z2); F- Deviation from z1; G-deviation from z2.); F-Deviation from z1; G-Deviation from z2

Out of the five problems,

a) 2 problems provided the better results than the approximate solution.
b) 2 problems provided the results equal to approximate solution.
c) 1 problem provided the results equal to lower bound value and approximate solution

6. Conclusion and Future Work

In this paper, a mathematical model is formulated and solution procedure is proposed for MSCN using RC-VPGA to find the optimum distribution system. In this work multi stage supply chain network with fixed charges incurred for operating each route is considered for the first time and solved by RC-VPGA.

The proposed methodology is evaluated for its solution quality by comparing it with the approximate and lower bound solutions. The comparison reveals that the RC-VPGA generates better solution than approximation method and is capable of providing solution closer to the lower bound solution of the problem. The paper is focused on the transportation cost as the single objective. The following are some future research directions in the field of multi stage supply chain network.

The time for transporting the products and the customer service levels may also be considered in the objective and the work may be extended. The present work considered only one product distribution. Multi commodity distribution may be attempted in future. The work may be attempted with other heuristics like Simulated Annealing, Ant Colony Optimization, Tabu search techniques and Particle Swarm Optimization.

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Nomenclature

cd	demand of the customer
Chr_len	length of the chromosome
c_{ij}	unit transportation cost of product flowing from supplier 'i' to plant 'j'
C_{ij}	equivalent transportation cost of product flowing from supplier 'i' to plant 'j'
c_{jk}	unit transportation cost of product flowing from plant 'j' to DC 'k'
C_{jk}	equivalent transportation cost of product flowing from plant 'j' to DC 'k'
c_{kl}	unit transportation cost of product flowing from DC 'k' to customer 'l'
C_{kl}	equivalent transportation cost of product flowing from DC 'k' to customer 'l'
DCC	capacity of DC
F	total new_fitness value of the population
f_{ij}	fixed charge associated with operating the route from supplier 'i' to plant 'j'
fit(C_i)	fitness value of chromosome
f_{jk}	fixed charge associated with operating the route from plant 'j' to DC 'k'
f_{kl}	fixed charge associated with operating the route from DC 'k' to customer 'l'
nc	number of customers
nd	number of distribution centers
new_fit(C_i)	modified fitness parameter of chromosome
np	number of plants
ns	number of suppliers
pc	capacity of the plant
p_c	probability of crossover
p_i	probability of selection of chromosome
p	probability of mutation
pop_size	size of the population
sc	capacity of the supplier
x_{ij}	units flowing from supplier 'i' to plant 'j'
x_{kl}	units flowing from DC 'k' to customer 'l'
x_{jk}	units flowing from plant 'j' to DC 'k'