



## WEIBULL ANALYSIS OF HIGH CYCLE FATIGUE INVESTIGATIONS OF CAST MAGNESIUM ALLOY UNDER TRANSVERSE LOAD

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### ABSTRACT

Magnesium is the lightest structural material used and it is important to explore its properties completely as it is used in aviation, automotive and communication. It is used as alloys combining with other metals. The behavior of the alloy at various ambiances has to be investigated. This paper reports the reliability analysis carried out on the high cycle fatigue investigations performed on cast AZ91 Magnesium alloys under transverse load. An Electro dynamic shaker system was used to apply transverse load on the specimens fabricated as per ASTM standard. Weibull analysis was under taken to check and compare the reliability of the investigations on gravity cast

**Keywords:** AZ91 Magnesium alloy, High cycle fatigue, Transverse loading, Weibull analysis.

### 1. Introduction

Magnesium is one of the lightest and abundantly available structural materials that are gaining popularity. It is used in defense, aeronautical, automobile, communication and many other sectors where weight reduction plays an important role. The weight reduction in the missiles, aircraft, automobile etc, increases their fuel carrying capacity there by increasing the operating distance. The weight reduction also indirectly reduces pollution. Among the many magnesium alloys AZ91 is an important Zirconium free magnesium alloy that serves in many fields.

About 80 to 90% of these structural materials, irrespective of the material experience due to time related fatigue failure caused by application cyclic load. The failure is catastrophic and happens well within the yield strength of the material. This type of failure cause severe damage to the equipments and cost loss of human life. Though the fatigue investigations are century old, even today the accident caused by fatigue failures do happen. This is due to lack of data in case of materials behavior under cyclic load. For example, most of the fatigue investigations are being carried out either in axial loading machines or in a rotating bending machine [1]. But the material's behavior under other environments cannot be predicted using these results. Hence, in order to predict a materials behavior under other loading environments new methodologies have to be developed. Transverse loading is another ambience apart from axial load whose effects requires to be

investigated completely. The transverse load is mostly caused by vibration which always exists in any structural elements.

To serve this purpose, a new testing methodology has been developed using an electro dynamic vibration shaker system which applies a transverse load to the specimen fabricated out of AZ91 Magnesium alloy as per ASTM standard D 671. The specimens were fabricated out of gravity cast pellets cast as per the conditions mentioned in one of the co-authors earlier works [2]. Gravity casting is widely used for mass production. The investigations were carried out under controlled lab atmosphere.

In practice, the patterns of failures overtime are often classified in to infant mortality, useful life and wear-out. These patterns can be recognized in mathematics by a combination of decreasing, constant, and increasing hazard functions. The three patterns combine to produce the bath-tube curve shown in fig 1. If the hazard function of the underlying distribution approximates to any part of the bathtub curve then the distribution is applicable as a time to failure model. Many Distributions including, uniform, normal, exponential, Rayleigh, Weibull, Erlang, Gamma, loglogistic, lognormal and others are often used in reliability analysis. Among these distribution, the Weibull distribution have decreasing hazard function for  $0 < \text{shape parameter} < 1$ , constant hazard function for shape parameter =1, and increasing hazard functions for shape parameter  $> 1$ . Since, it suits to all the three

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hazard functions, it is chosen for the analysis. Results obtained from the investigations were theoretically analyzed using Weibull analysis

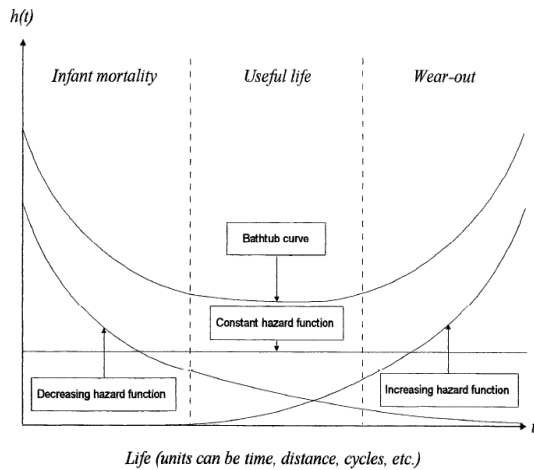


Fig. 1 Bathtub Curve

## 2. Experimental Procedure

### 2.1 Materials and equipments

Specimens (Fig 2) of constant length (57.2 mm) and thickness (4 mm) with varying cross sectional area are fabricated as per ASTM std D 671. Fully reversed, (Stress ratio = -1) constant amplitude of transverse load is applied to small end of the specimen using an electro dynamic vibration shaker system at a controlled laboratory temperature of 24°C. The electro dynamic shaker system consists of a signal conditioner, power amplifier and a shaker unit. The output of the signal conditioner is amplified in the power amplifier to drive the shaker, which converts the amplified signal to mechanical motion at the worktable. The movement of the shaker table is sensed by a piezo electric accelerometer for monitoring and operational control. The mounting fixtures and the supporting structures are fabricated specially to suit the current investigation.

### 2.2 Experimental procedure

The testing frequency of 50Hz is decided on calculating the natural frequency of the specimen as reported by Thomson & Marie [3] to avoid occurrence of resonance. The fabricated specimens were loaded in sequence on the table, in such a way that movement of bigger end specimen is constrained completely and the small end is loaded to move in a transverse direction (Fig 3). The maximum load and the applied load are calculated using the Navier’s flexure formula. The deflection for the maximum load and the applied load

was determined using the equation (1) for maximum deflection of a cantilever

$$Y_{\max} = PL^3/3EI \tag{1}$$

Where  $Y_{\max}$  is the transverse deflection; P is the load and L is the test length and E is the Young’s modulus. The acceleration of the shaker system is checked using the equation (2)

$$A = 4\pi^2f^2Y/9810 \tag{2}$$

Where ‘f’ is the testing frequency and Y is the deflection. Hence, by changing the deflection of the specimen in the transverse direction, the load can be varied and hence the stress. The cyclic load is applied in the decided frequency earlier till it fails or it is cycled upto  $10^7$  number of cycle. The stress and the number of cycles for each specimen is noted and the S-N plot for the transverse loaded gravity cast AZ91 magnesium alloy was generated. (Fig 4)

Semi log scatter plot of the Stress and the number of cycles for failure was generated from the test results. There were few runouts during the test. The runout specimens were indicated using arrow heads in the S-N plot. The endurance limit of the tested alloy under transverse load was ascertained from the generated S-N plot, indicates deviation of the endurance limit of the same alloy tested under axial load. Theoretical analysis of the test results were carried out using Weibull distribution.

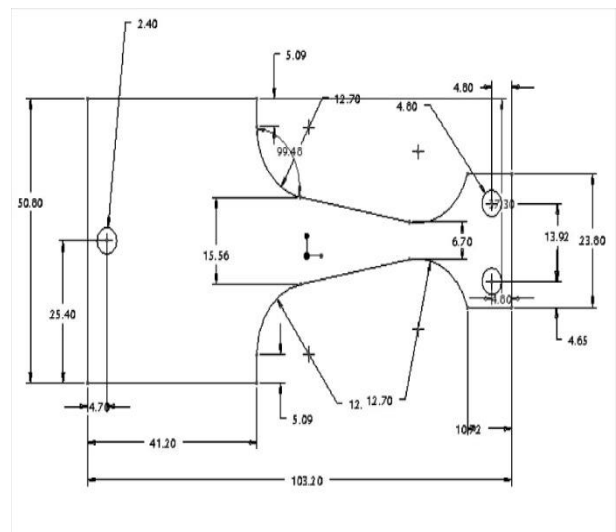


Fig. 2 Specimen as per ASTM STD D671

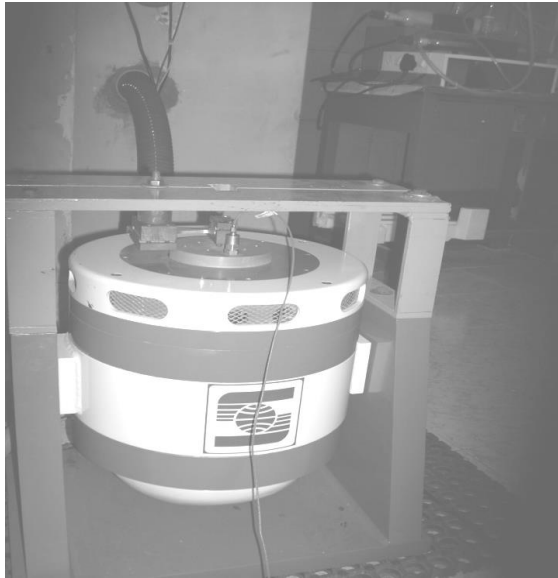


Fig. 3 Specimen Mounting on the Shaker Table

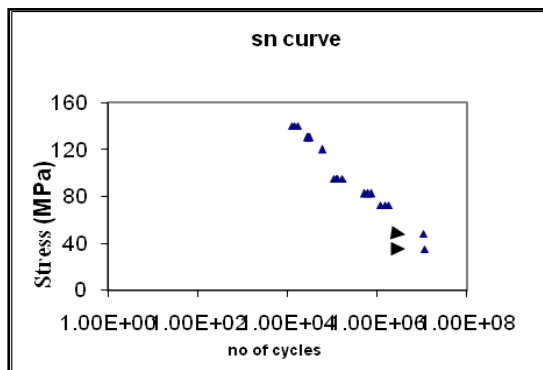


Fig. 4 S-N Plot of Gravity Cast AZ91 Alloy

### 3. Theoretical Analysis

#### 3.1 Weibull distribution

Goodness of fit tests is used to determine the best fit among the four different plots among the Weibull, extreme value, exponential, normal, long normal, logistic and loglogistic distribution. Tests using Anderson-Darling for the maximum like hood and least squares estimation methods and Pearson Correlation coefficient for the least squares estimation method help to assess how the distribution fits to the chosen data. The Anderson-Darling statistic is a measure of how far the plot points fall from the fitted line in a probability plot. A smaller Anderson Darling statistic indicates that the distribution fits the data better. For least squares estimation, a Pearson correlation coefficient is used. If the distribution fits the data well, the plot points on a

probability plot will fall on a straight line. The correlation measures the strength of the linear relationship between the X and Y variables on a probability plot. The correlation will range between 0 and 1 and higher values indicate a better fitting distribution. From the Goodness of fit tests, undertaken for the chosen data, Weibull distribution fits in a better way than other distributions

There are three major ways which are used to express the scale and shape parameters of Weibull distribution (W). They are  $W(\eta, \beta)$ ,  $W(\eta^{-\beta}, \beta)$  and  $W(\eta^{\beta}, \beta)$ . The Weibull model has been widely used in reliability analysis. It has been successful in modelling life of many devices and variables like relays, ball bearings, electron tubes, capacitors, germanium transistors, photo-conductive cells, motors, automotive radiators, regulators, generators, turbine blades, fatigue in textiles, corrosion resistance, leakage of dry batteries, return of products after-shipment, marketing life expectancy of drugs, the number of downtimes per shift, solids subjected to fatigue stresses and many other such applications.

The cumulative distribution function of the three parameter Weibull distribution is given as follows:

$$F_{(s;a,b,c)} = 1 - \exp\left(-\left(\frac{x-a}{b}\right)^c\right) \quad (3)$$

Where a,b,c are the location, scale and shape parameters respectively. When a = 0 in equation (3)

The parameters b and c of the distribution function  $F(x;b,c)$  are estimated from observations. The methods usually employed in the estimation of these parameters are methods of linear regression, methods of maximum like hood, and method of moments. Among these methods, linear regression method is used. This method is based on transforming equation (4) into following form.

$$F_{(x;b,c)} = 1 - \exp\left(-\left(\frac{x}{b}\right)^c\right), \quad b \geq 0, c \geq 0 \quad (4)$$

$$1 - F_{(x;b,c)} = \exp\left(-\left(\frac{x}{b}\right)^c\right) \quad (5)$$

And linear regression model is obtained by transforming equation (5) to the form  $Y = mX + \gamma$ .

$$\ln\left[\ln\left(\frac{1}{1 - F_{(x;b,c)}}\right)\right] = c \ln(x) - c \ln(b) \quad (6)$$

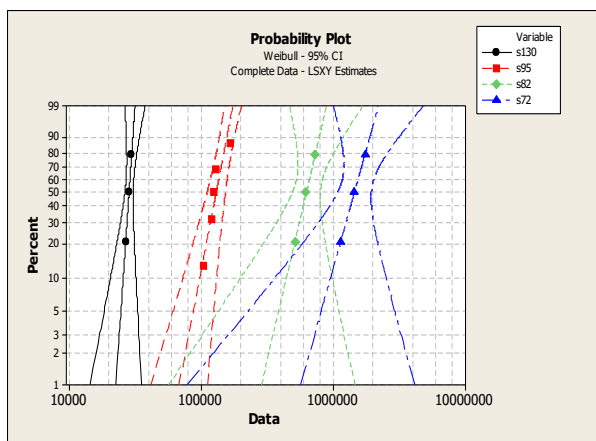
The two parameter Weibull distribution is defined by two parameters: shape and scale. The shape describes the shape of the Weibull curve. A shape of 3 approximates a normal curve. A shape between 2 and 4

is still fairly normal. A low value for shape, say 1.25, gives a right-skewed curve. A high value for shape, say 10, gives a left-skewed curve. The scale, or characteristic life, is the 63.2 percentile of the data. The scale defines the position of the Weibull curve relative to the threshold, analogous to the way the mean defines the position of a normal curve. The threshold is a shift of the distribution away from 0. A negative threshold 0, and a positive threshold shifts the distribution to the right of 0. All data must be greater than the threshold.

**3.2 Distribution analysis**

Minitab R 14 software was used to carryout the Goodness of fit tests and distribution analysis. The distribution displays common measures of the center and spread of the distribution with 95% lower and upper confidence intervals. The mean and the standard deviation are not resistant to large life times while the median, 75<sup>th</sup> percentile and the inter quartile range are resistant. The values in the percentile column of the results estimates the times at which the corresponding percent of the units fails.

For the initial investigation, the samples tested in the test ranging from 130 MPa to 72 MPa having a total sample size of 14 nos. Weibull probability plots were constructed from the sample data. The plot consists of plot points which represent the proportion of failures up to a certain time. The fitted line which is a graphical representation of the percentiles. To make the fitted line, various percentile were estimated for the various percents based on the chosen distribution. The associated probabilities are then transformed and used are y variables. The present investigation shows the points hug close to the center line, showing the best fit to the chosen Weibull distribution in all the four selected stress value.



**Fig. 5 Probability Plots for Various Stress Values**

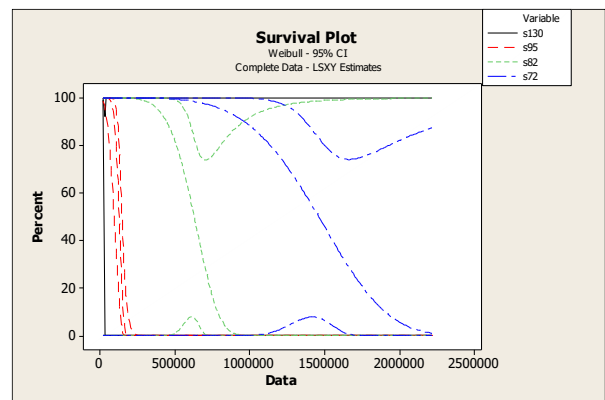
The results probability plots of different stress is shown in the Table1.

**Table 1: Results – Probability Plot**

Probability	130 MPa	95MPa	82MPa	72MPa
1%	22727	68635	289345	574770
50 %	28616	130905	631595	1462835
99 %	31723	174736	895535	2221591

These probability plot (Table 1) results correlate well with the test results of the gravity cast AZ91 alloy under transverse load. It is also inferred that the results have 98 % correlation and increases as the stress level decreases.

The survival plot (Fig. 6) displays the survival probabilities versus data. The resultant survival plot results displays that there is no probability of survival above 138,457 cycles at the stress of 130 MPa. The number of cycles for survival keeps on increasing as the stress level lowers. This also correlates with the tested results. The deviation in the survival plot indicates the scatter in the tested results which are caused by the inbuilt casting defects or pores or poor surface condition of the individual specimen.



**Fig. 6 Survival Plot for Various Stress Values**

The cumulative distributive function (CDF) plot is generated for the various stresses using the tested results. The cumulative distribution of the test results under a environment. The results reveal that the CDF plot is more intuitive .Only one percent of specimen can fail up to 55,307 cycles and 99% of the specimen will fail around 2318125 at a stress value of 72 MPa.

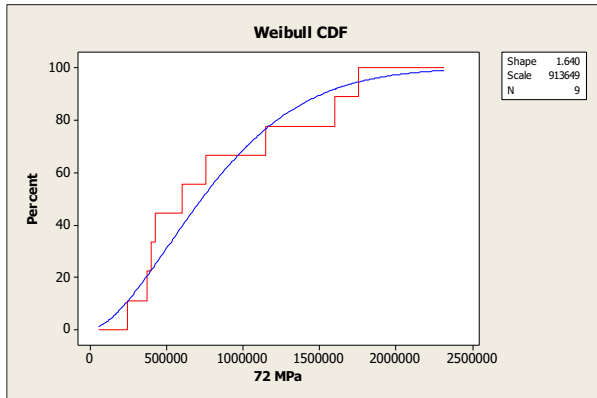


Fig. 7 CDF Plot for a Stress Value (72 MPa)

#### 4. Conclusions

High cycle fatigue investigations were carried out using gravity cast AZ 91 Magnesium alloy under transverse load using electro dynamic shaker system. From the goodness of fits tests conducted it was ascertained that the Weibull distribution suits for

theoretical analysis of the test results. From the initial investigations conducted at the higher stress levels, it is found that the results obtained from the Weibull distribution, correlates well with the results of test conducted.

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