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WAVELET DE-NOISING USING CUSTOMIZED THRESHOLDING FOR BEARING FAULT DETECTION

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ABSTRACT

Denoising is the process of recovering the original signal from the signal corrupted by noise. This problem of denoising has been topics of research.The method based on the wavelets have been the topic for research. In this paper, multiresolution analysis is applied to de-noise a simulated signal and signal obtained from a defective bearing. In this work, the characteristic features of vibration signals are extracted from noise Daubechies wavelets. Thresholding is one of the most commonly used processing tools in wavelet signal processing for noise removal. The methods used for estimating the threshold values are Rigrsure, Sqtwolog, Heusure and minimax. The two versions of thresholding a signal which are used to reduce the effect of noise are soft thresholding and hard thresholding. The propose technique called the customized thresholding function, is a linear combination of the soft and hard thresholding. Comparison of the new method with the existing thresholding methods is provided. Simulation results and the application on the actual signal demonstrate the advantage of using this method. Signal-to-noise ratio (SNR) and Mean Square Error (MSE) is used for comparing the use of wavelets and denoising techniques.

Keywords: *Wavelet Analysis, Denoising, Thresholding, Fault Detection*

1. Introduction

Early fault detection in bearings will save time and reduce the economical loss. The damage in the bearing may be due to loading condition, operating condition or may be during mounting the bearing. Damage of these usually causes the vibration level of the system to increase. Early detection of the minor damage is more essential. Vibration analysis is one of the popular techniques used in the detection of the damage. Displacement transducer, velocity pick-up or an accelerometer is used to pick–up the signals. Signals transmitted not only contain the signals due to vibration but also vibrations from the meshing gears and the other running parts. Considering the environment in which they are generated, vibration signals are very noisy. This noise must be removed to correctly evaluate the signals. Hence, noise removal from the collected signals is an important step in the effective fault detection. The reduction of the noise in the signals improves the Signal-to-noise (SNR) ratio. Wavelet de-noising technique can be used for this purpose.

The problem of estimating an unknown signal embedded in Gaussian noise has received a great deal of attention in numerous studies. The denoising process is used to separate an observed data sequence into a "meaningful" signal and a remaining noise. We want the recovered signal to be as close as possible to the original signal, retaining most of its important properties (e.g. smoothness). Traditional denoising schemes are based on linear methods, where the most common choice is the Wiener filtering. Recently, nonlinear methods, especially those based on wavelets have become increasingly popular. One of the earliest papers published in the field of wavelet-based denoising was by Weaver et.al. [1]. In this pioneering work, they proposed a new method for filtering noise from MR (Magnetic Resonance) images based on the hard-thresholding scheme. They showed that by using wavelet-thresholding, the noise could be significantly reduced without reducing the edge sharpness. The advantages of the wavelet denoising scheme presented by Weaver, et al. were mainly based on experimental results. Donoho and Johnstone proved several important theoretical results on wavelet thresholding, or wavelet shrinkage [2, 3]. They showed that wavelet shrinkage has many excellent properties, such as near optimality in minimax sense, and a better rate of convergence. Thresholding is one of the steps in

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wavelet denoising. Reconstructed signal depends upon the thresholding level.

In this paper, a new thresholding function is proposed that can take the place of the traditional thresholding functions, such as soft-thresholding and hard-thresholding. The results obtained from the simulation and the application of this technique on the actual signal obtained using the experimental set-up show that the proposed method is advantageous and helps in improving the denoised results significantly.

2. Wavelet De-Noising

The removal of noise from noisy data to obtain the signal of interest is often referred to as denoising. The method of signal denoising via wavelet thresholding was popularized by Dohono et al. [2,3].In signal denoising a compromise has to be made between noise reduction and preserving significant signal details. Wavelet Transform implements both low-pass and high-pass filters to the signal. The low-frequency parts reflect the signal information, and the high-frequency parts reflect the noise and the signal details. The underlying model for the noisy signal is basically of the following form

$$S(t) = f(t) + \sigma\varepsilon(n) \tag{1}$$

Where, time t is equally spaced. In the simplest model we suppose that $\varepsilon(n)$ is a Gaussian white noise with mean 0 and standard deviation σ. The de-noising objective is to suppress the noise part of the signal $S(t)$ and to recover the signal $f(t)$. The method does involve the shrinkage in the wavelet domain which results in an overall reduction in size of the wavelet coefficients which will reduce the coefficients of negligible value to zero. The presence of noise in the signal affects all the coefficients regardless of the scale. Shrinking them towards zero has the effect of suppressing the noise while preserving the initial features of the signal.

Denoising with wavelet consists of three steps: [3, 4, 5, 6].

- Wavelet Decomposition. Transform the noisy data into wavelet domain
- Wavelet Thresholding. Apply soft or hard thresholding to the high-frequency coefficients; thereby suppress those coefficients smaller than certain amplitude. Thresholding to the decomposed high-frequency coefficients on each level can effectively denoise the signal
- Reconstruction. Transform back into the original domain

In the whole process, a suiTable wavelet, an optimal decomposition level for the hierarchy and one appropriate thresholding function should be considered (Mallat 1999). But the choice of threshold is the most critical.

2.1 Thresholding parameters

Thresholding reduces the effect of the noise without changing the effect of the signal. The thresholding is based on a value that is used to compare with all the detailed coefficients. Two popular versions of thresholding a signal are soft thresholding and hard thresholding. The definitions of the two methods of thresholding are given below.

Hard thresholding: The wavelet coefficient is retained if its value is more than the threshold value. It sets any coefficient less than or equal to the threshold, to zero.

$$f_h(x) = \begin{cases} x & if\,|x| \geq \lambda \\ 0 & otherwise \end{cases} \tag{2}$$

Soft thresholding: It sets any coefficient less than or equal to the threshold to zero. The threshold is subtracted from any coefficient that is greater than the threshold. This moves the time series toward zero.

$$f_c(x) = \begin{cases} sign(x)(|x| - \lambda) & if\,|x| \geq \lambda \\ 0 & otherwise \end{cases} \tag{3}$$

Dohono et al. [2,3] introduced the following four threshold value determination and are available in MATLAB wavelet toolbox [7]. We have considered four threshold selection rules: [8, 9]

Rigrsure: Threshold is selected using the principle of Stein's Unbiased Risk Estimate (SURE), which has a quadrature loss function. We get an estimate of the risk for a particular threshold value. Minimizing the risks gives a selection of the threshold value.

Sqtwolog: This is a fixed form threshold yielding minimax performance multiplied by a small factor proportional to log (length(s)), where s = s(x) is the signal to be denoised. It is usually equal to sqrt (2* log (length (s))

Heursure: Threshold is selected using a mixture of the first two methods. If the signal-to-noise ratio is very small, the SURE estimate is very noisy. So if such a situation is detected, the fixed form threshold is used.

Minimaxi: Threshold is selected using the minimax principle. This uses a fixed threshold chosen to yield minimax performance for mean squared error against an ideal procedure. The minimax principle is used in statistics to design estimators. Since the denoised signal can be assimilated to the estimator of the unknown regression function, the minimax estimator is the option that realizes the minimum, over a given set of functions, of the maximum mean squared error.

The algorithm of the threshold selection is,

$$\text{Threshold } \lambda = 0.3936 + 0.1829 \log_2(n) \tag{4}$$

3. Simulation

A valid signal model is necessary for accurate vibration detection. As given in equation (1), a vibration waveform is represented as a sinusoid and normal distribution white noise (Gaussian noise).

$$S(t) = f(t) + \sigma\varepsilon(n) \tag{5}$$

To better approximate the actual vibration environment, the vibration model will be given as,

$$S = \sin(2\pi f_1 t) + 2\sin(2\pi f_2 t) + randn(t) \tag{6}$$

Where t=0,0.001,0.002,…2 *s*, f_1=25 Hz, f_2=80 Hz and for this model the vibration signal is totally corrupted by additive white noise, represented by *randn(t)*.

Fig. 1 shows the flow chart for wavelet based vibration detection program. The above signal was denoised using soft thresholding. Rigrsure (R), Sqtwolog(S), Heusure (H) and Minimax (M) methods were used to determine the threshold value. SNR and Retained Energy are used to compare the methods. There are many functions available that can be used as a mother wavelet, such as Haar, Daubechies, Meyer and Morlet function [10, 11].

In this work, the signal is decomposed at level 5 using some DbN wavelets. Table 1 and Table 2 show the SNR values and the retained signal energy at 25%, 50% and 75% of the threshold value respectively. It is clear from the analysis that among the four thresholds selection rules used, the Rigrsure (SURE) and Minimax are more conservative than others. On the basis both SNR and retained energy, Db4 and Db8 give a better performance hence further study is based on Db8 as mother wavelet.

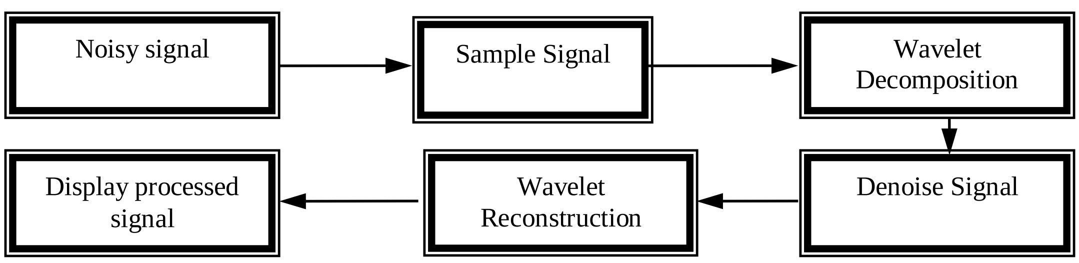

Fig. 1 Flowchart of the Wavelet-Based Vibration Detection Program

Table 1: SNR Values at 25%, 50% and 75% of the Threshold Value

SNR	25 %				50 %				75 %			
	R	**S**	**H**	**M**	**R**	**S**	**H**	**M**	**R**	**S**	**H**	**M**
Db4	10.249	6.274	7.487	10.043	6.168	2.777	4.629	5.427	4.740	1.474	4.075	3.425
Db6	9.7010	6.314	7.227	10.065	5.930	2.775	4.204	5.464	4.727	1.512	3.589	3.430
Db8	10.130	6.318	7.257	10.098	5.923	2.825	4.309	5.429	4.516	1.597	3.781	3.432
Db10	8.5040	6.415	7.374	10.171	5.116	2.891	4.451	5.551	4.167	1.570	3.829	3.549
Db11	9.5540	6.407	7.351	10.147	5.785	2.897	4.403	5.551	4.159	1.596	3.805	3.559
Db12	9.2340	6.337	7.295	10.059	5.463	2.821	4.346	5.480	4.297	1.617	3.852	3.460
Db13	9.7730	6.332	7.266	10.047	4.722	2.840	4.365	5.476	4.320	1.605	3.861	3.474
Db14	8.7450	6.353	7.299	10.062	5.128	2.866	4.398	5.503	4.116	1.600	3.880	3.502

Table 2: Retained Signal Energy at 25%, 50% and 75% of the Threshold Value

Retained Energy	25 %				50 %				75%			
	R	S	H	M	R	S	H	M	R	S	H	M
Db4	71.05	42.06	64.13	60.60	57.39	19.89	53.78	38.20	50.46	10.47	49.16	25.44
Db6	71.30	41.99	62.50	60.61	59.39	19.64	52.39	38.08	53.92	10.10	49.13	25.17
Db8	71.77	42.41	63.81	60.83	59.51	20.63	55.00	38.52	54.49	11.05	52.35	26.01
Db10	67.85	42.97	64.36	61.30	57.04	20.72	55.00	39.05	52.89	11.09	52.01	26.21
Db11	70.49	43.16	63.79	61.34	58.43	21.00	54.08	39.23	53.03	11.43	50.85	26.45
Db12	69.60	42.70	63.99	60.94	57.93	20.80	54.79	38.74	53.35	11.40	51.87	26.10
Db13	70.42	42.70	64.02	60.94	57.88	20.70	54.93	38.74	52.76	11.21	51.99	26.06
Db14	67.93	42.92	64.08	61.07	56.47	20.90	54.83	38.96	52.14	11.35	51.72	26.28

4. Customized Thresholding

The denoising algorithms, which are based on thresholding, suggest that each coefficient of every detail subband is compared to a threshold level λ and is retained if the coefficient is greater than the threshold value or equated to zero if it is less the threshold value. Fig. 2 indicates the two types of thresholding.

The hard type does not affect the coefficients that are greater than the threshold level, whereas the soft thresholding causes the shrinkage of these coefficients. Note that the hard thresholding function is discontinuous at $|x| = \lambda$ and due to this function yields abrupt artifacts in the denoised signal especially when the noise level is significant [6].The above disadvantage may be slightly overcome by using the customized thresholding function. The function is a linear combination of the hard thresholding and the soft thresholding function. The functions are expressed as Eqn. (7) and Eqn. (8).

$$f_c(x) = (1-a)\, f_h(x) + a f_s(x) \qquad (7)$$

Using Eqn.(2) and Eqn.(3) in Eqn.(7), we can write custom thresholding function as,

$$f_c(x) = \begin{cases} (1-a)x + a(sign(x)(|x| - \alpha\lambda)) & if\ |x| \ge \lambda \\ 0 & otherwise \end{cases} \qquad (8)$$

Here, variable '*a*' decides the shape of the function and α is the percentage of the threshold λ applied. Fig. 3 depicts the thresholding function for different values of *a* when α =1 and λ=1.Also, $\underset{a\to 0}{f_c(x)} = f_h(x)$ and $\underset{a\to 1}{f_c(x)} = f_s(x)$ which shows that it can also be used for hard and soft thresholding.

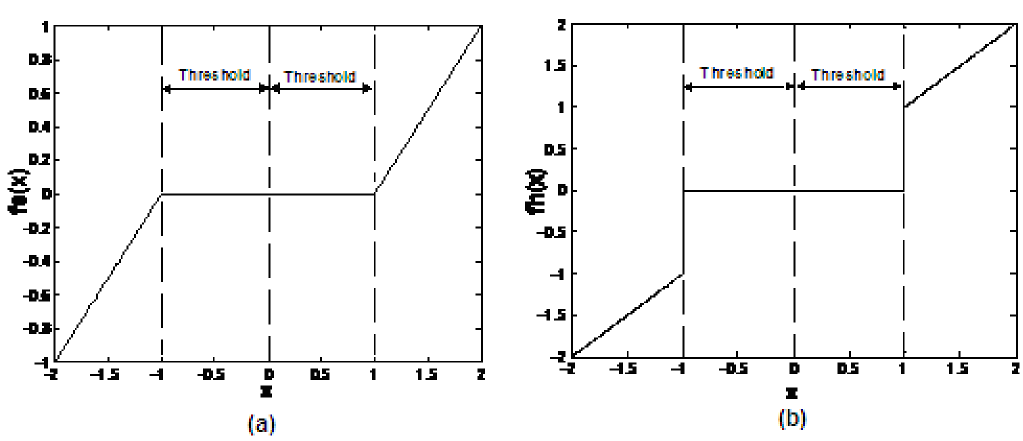

Fig. 2 Threshold Types (a) Soft Thresholding (b) Hard Thresholding

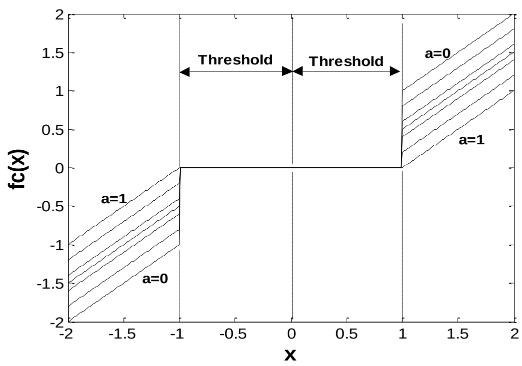

Fig. 3 Custom Thresholding Function for Different Values of *a*

5. Results of Customised Thresholding

The customized thresholding function was used for denoising the above simulated signal. The performance parameters SNR, retained signal energy and MSE were determined for *a*= 0.2 to 1.0 at interval of 0.2 and the thresholding percentage α= 25%, 50% and 75%. *a* =1 correspond to soft thresholding. Results are obtained using Db8 wavelet and are shown in Tables 3-5. It is seen that thresholding at *a* = 0.2 to 0.6and using Rigrsure and Minimax method give good results. The Fig.4 shows the result of custom thresholding on the simulated signal using Rigrsure method (α = 0.7 and *a* = 0.4).

Table 3: SNR Values using Custom Thresholding

SNR	α = 25 %				50 %				75 %			
	R	S	H	M	R	S	H	M	R	S	H	M
a = 0.2	16.496	11.753	12.106	17.445	9.064	4.999	5.515	10.066	5.726	3.141	4.151	5.916
a = 0.4	14.983	10.573	11.112	15.531	8.524	4.660	5.345	9.136	5.555	2.915	4.103	5.517
a = 0.6	13.252	9.123	9.838	13.507	7.750	4.148	5.075	7.927	5.284	2.563	4.024	4.923
a = 0.8	11.607	7.671	8.516	11.684	6.855	3.519	4.723	6.655	4.931	2.114	3.916	4.027
a = 1.0	10.130	6.318	7.254	10.098	5.923	2.825	4.309	5.429	4.516	1.597	3.781	3.432

Table 4: Retained Signal Energy Values using Custom Thresholding

Retained Energy	25 %				50 %				75 %			
	R	S	H	M	R	S	H	M	R	S	H	M
a = 0.2	91.55	80.86	86.14	89.64	80.22	56.03	67.34	77.26	68.34	40.56	59.27	62.01
a = 0.4	85.67	69.17	78.98	81.45	73.40	44.61	63.13	65.22	63.80	30.60	57.11	50.56
a = 0.6	80.42	58.86	72.87	73.91	67.67	34.91	59.67	54.76	59.98	22.36	55.24	40.71
a = 0.8	75.78	49.94	62.81	67.04	63.04	26.92	56.96	45.85	56.87	15.84	53.65	32.54
a = 1.0	71.77	42.41	63.81	60.83	59.51	20.63	55.00	38.52	54.49	11.05	52.35	26.01

Table 5: MSE using Custom Thresholding

MSE	25 %				50 %				75 %			
	R	S	H	M	R	S	H	M	R	S	H	M
a = 0.2	0.045	0.135	0.124	0.036	0.250	0.637	0.566	0.199	0.539	0.978	0.775	0.516
a = 0.4	0.064	0.177	0.156	0.056	0.283	0.689	0.589	0.246	0.561	1.030	0.783	0.566
a = 0.6	0.095	0.247	0.209	0.090	0.338	0.775	0.626	0.325	0.597	1.117	0.798	0.649
a = 0.8	0.139	0.345	0.284	0.137	0.416	0.896	0.679	0.435	0.647	1.239	0.818	0.765
a = 1.0	0.196	0.471	0.379	0.197	0.515	1.051	0.747	0.577	0.712	1.395	0.844	0.914

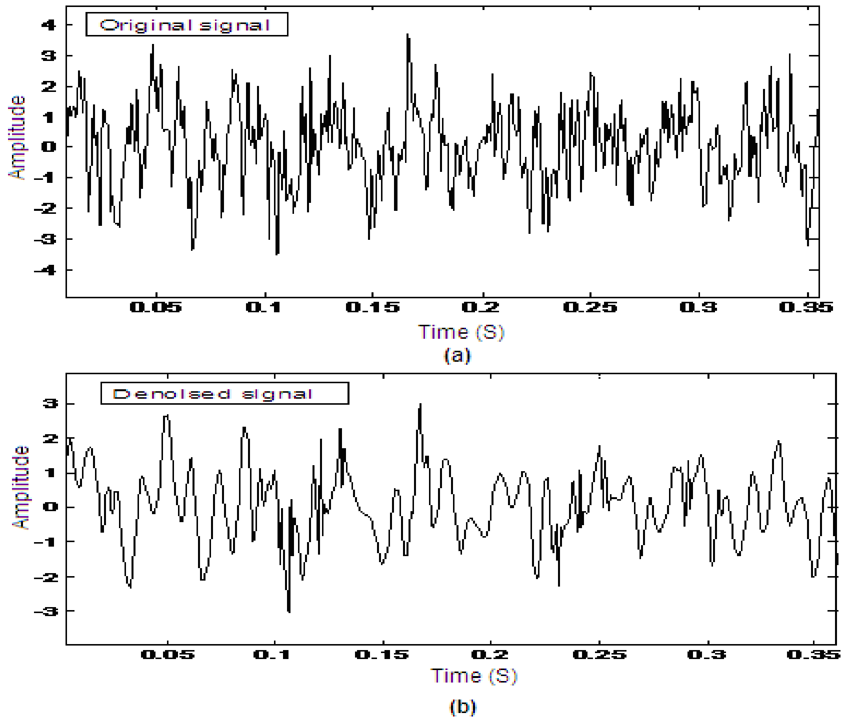

Fig. 4 Simulated Signal (a) With Noise (b) Denoised using Custom Thresholding

Now we consider the fault detection in the ball bearing. Two bearings, SKF 6305, were tested for comparison. One was the normal bearing and the other having a defect induced on the outer race. The defect was induced using an EDM. An accelerometer was used to pick up the signal. The shaft speed was 1400 rpm and the bearing is subjected to a load of 1.5 KN. Each bearing has 7 balls, ball diameter = 11.5 mm, pitch diameter= 43.5 mm and contact angle =0 (assumed). Fig.5 and Fig. 6 show the actual and the denoised vibration signal of normal bearing and defective bearing respectively. Both the stages can be easily distinguished. The characteristic defect frequency is found to be 60.07 Hz and the period is 0.0166 seconds. The period was identified approximately as 0.018 seconds.

6. Conclusion

It is seen from the results that the custom thresholding proposed in this paper outperforms the traditional soft thresholding schemes and hence can be used as one of the methods for denoising. The advantage of this function is the shape of the thresholding function and the percentage of thresholding can be adapted to the characteristics of the given signal, resulting in a smaller estimation error.

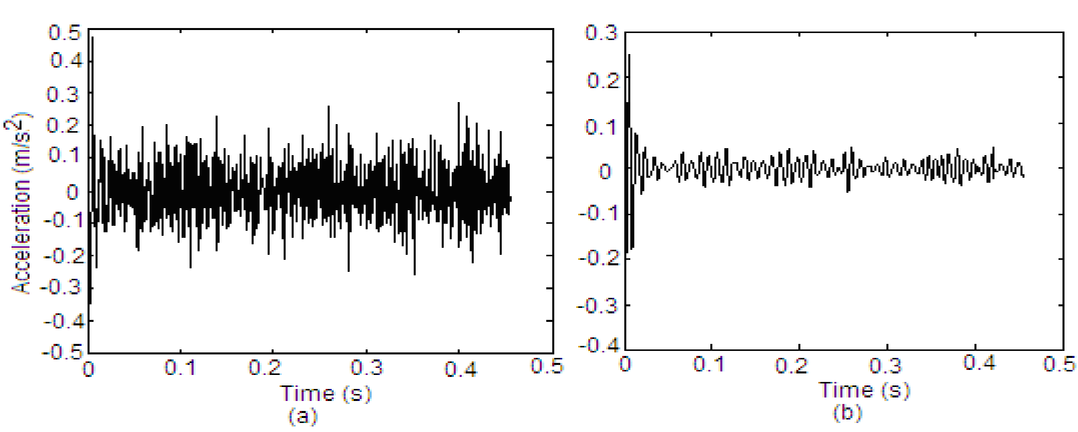

Fig. 5 Signal from Normal Bearing (a) Original (b) Denoised

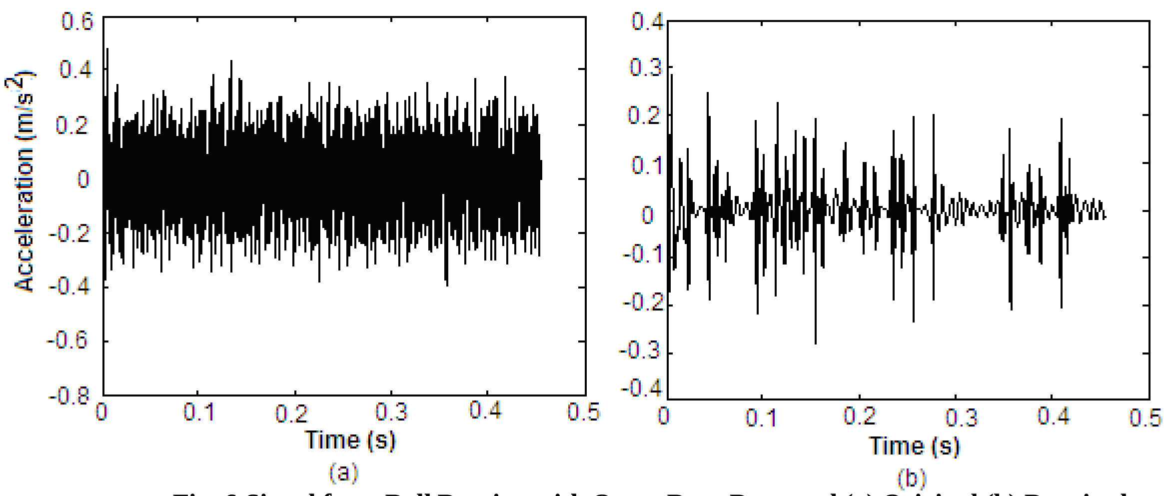

Fig. 6 Signal from Ball Bearing with Outer Race Damaged (a) Original (b) Denoised

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Nomenclature

Symbol	Description	Units
a	Factor that decides the shape of the function	
$f(t)$	Original signal	
f_1,f_2	Frequency	Hz
t	Time	s
$S(t)$	Noisy Signal	
x	Value of wavelet coefficient	
α	percentage of the threshold	
σ	Standard Deviation of noise	
λ	Threshold value	