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DEVELOPMENT OF MATHEMATICAL MODEL FOR ANALYSIS OF CYLINDRICAL COMPONENTS MADE OF FUNCTIONALLY GRADED MATERIALS UNDER PRESSURE AND THERMO MECHANICAL LOADING

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ABSTRACT

The gradual linear variation in properties of the functionally graded circular hollow cylinder is assumed to be split up into multiple layers in the radial direction and each of which is individually isotropic in nature. The cylinder is assumed to be of finite length and is subjected to axisymmetric thermal and mechanical loads. The fundamental thermo-mechanical relationships obtained for the two-layer isotropic material is extended to the assumed concept of multiple layer isotropic material applicable to functionally graded material. The solutions for temperature, displacements, and thermal/mechanical stress distribution in a functionally graded circular hollow cylinder are presented in this paper. The mathematical model which is developed can be used to analyze functionally graded circular hollow cylinder.

Keywords: *Functionally graded circular hollow cylinder, Thermo-mechanical loading, Multi-layered approach*

1. Introduction

Functionally Graded Materials (FGMs) are new materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to other. FGMs were initially designed as thermal barrier materials for aerospace structural applications and fusion reactors. These materials are now developed for general use as structural elements in extremely high-temperature environments. There has been increasing use of the FGMs as structural elements for space and industrial applications.

Stanley and Chau [1] studied the failure probability analysis for a RBSN (Reaction Bonded Silicon Nitride) cylinder with a SiC (Silicon Carbide) coating subjected to a radial heat flux applying the design methodology to bodies consisting of two different isotropic layers under steady state thermal loads. This work explains that the application of coating to components subjected to thermal stresses due to temperature gradients offers the possibility of reductions in stress levels.

Literature is abundant with the materials dealing with the manufacturability and characterization of the FGMs. Various powder processing technologies viz. backward extrusion of $Al\text{-}Al_3$ Ti platelet particles by a centrifugal solid-particle method [2], a new solid free form fabrication technology, namely, laser rapid forming (LRF) to fabricate bulk near-net-shape metallic based components [3], centrifugal casting technique to obtain materials with higher density on the outer regions of a casting due to applied centrifugal forces [4], Laser cladding (LC)-based freeform fabrication technology which forms strongly bonded layers of fully dense and possibly homogeneous structures [5], multi-directional laser-based direct metal deposition, an additive manufacturing process to get the desired shape and orientation of the volume fraction of the constituent materials [6] are in use to produce the FGMs with the desired shape and orientation of the volume fraction of the constituent materials.

The process capability to produce the FGMs with the tailored property variations in the chosen direction by smoothly varying the volume fraction of the constituent materials has enabled researchers with the option to define the variations in the properties in a particular direction by the mathematical equation models they assume. Patel et al.[7] chose the property variation model of a non-circular cylindrical shell, functionally graded in the radial direction. Sofiyev [8] choose the effective material properties varying in axial direction according to rule of mixtures. Esalmi et al. [9] assume the sphere’s material is assumed to be described

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with a power law function of the radial coordinate variable in their study of thermal and mechanical stresses in functionally graded thick sphere. Ferreira et al. [10] use the homogenization techniques to find the effective property at a point according to rule of mixtures. Ootao and Tanigawa [11] assume the material properties vary in the radial direction using the power law functions in their study of thermo-elastic analysis of a functionally graded cylindrical panel. Marin [12] list out the fields of applications of the FGMs as thermal barrier coatings for space applications, nuclear fast breeder reactors etc.

Jin et al. [13] specify the suitability of the mullite/Mo FGM in thermal barrier materials because of the constituents with similar thermal expansion coefficients and so are bound to have minimal thermal expansion mismatch stresses under very high thermal loads.

Shao [14] studies the steady-state thermal/mechanical stresses of a functionally graded circular hollow cylinder with finite length.

This paper deals with the mathematical model to analyse steady-state thermal and mechanical behaviour of a functionally graded circular hollow cylinder of finite length. The cylinder is assumed to be composed of N equidistant infinitesimal layers of isotropic nature.

The equations for temperature distributions, radial and axial displacements and stresses in the three directions obtained for two layer compound cylinder is extended to the multi-layered concept at the interfaces of the layers of the functionally graded circular hollow cylinder.

2. Equations for Two Layers

The fundamental equations for temperature distribution and stress-strain relationships are first obtained for a cylindrical component made up of two layer isotropic material, for various thermal boundary specific conditions [1].

A compound cylinder made up of two layers is considered. Cylinder inner radius *a*, outer radius c and interfacial radius *b* is shown in Fig. 1. The cylinder with finite length L is subjected to steady-state temperature loads T_{in} at inner surface, T_{out} at outer surface. The cylinder is also subjected to internal pressure p_a and external pressure p_c or atmospheric pressure. The equations for stresses developed and radial and axial displacements are obtained.

After determining the temperature distributions in the layer 1 and layer 2, the thermal stresses are calculated as follows

2.1 Stress-temperature relations

For layer 1,

$$\sigma_{r1} = \frac{E_1\alpha_1}{1-\upsilon_1}\frac{1}{r^2}\left\{\frac{r^2-a^2}{b^2-a^2}\int_a^b T_1 r dr - \int_a^r T_1 r dr\right\} \quad (1)$$

$$\sigma_{\theta 1} = \frac{E_1\alpha_1}{1-\upsilon_1}\frac{1}{r^2}\left\{\frac{r^2+a^2}{b^2-a^2}\int_a^b T_1 r dr + \int_a^r T_1 r dr - T_1 r^2\right\} \quad (2)$$

$$\sigma_{z1} = \frac{E_1\alpha_1}{1-\upsilon_1}\left\{\frac{2}{b^2-a^2}\int_a^b T_1 r dr - T_1\right\} \quad (3)$$

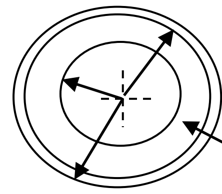

Fig. 1 Cross-Section of the Two–Layer Cylindrical Model

Where subscripts r, θ, z refers to the radial, hoop and axial directions respectively. E, α, ν and T refer to the Young's modulus, thermal expansion co-efficient, Poisson's ratio and temperature distribution in the layer respectively. Subscripts 1 and 2 refer to layer 1 and layer 2 respectively.

For the stresses in the layer 2, 'a' is replaced by 'b' and 'b' by 'c'. Also E_1, υ_1 and α_1 by E_2, υ_2 and α_2 respectively and T_1 by T_2 in equations (1), (2) and (3).

2.2 Radial and axial displacements at the interface

Due to temperature gradient at the interface, (r = b),

$$u_{t1} = b\varepsilon_{\theta 1(r=b)} = \frac{2b\alpha_1}{b^2-a^2}\int_a^b T_1 r dr, \quad w_{t1} = L\varepsilon_{z1} = \frac{2L\alpha_1}{b^2-a^2}\int_a^b T_1 r dr \quad (4)$$

$$u_{t2} = b\varepsilon_{\theta 2(r=b)} = \frac{2b\alpha_2}{c^2-b^2}\int_b^c T_2 r dr, \quad w_{t2} = L\varepsilon_{z2} = \frac{2L\alpha_2}{c^2-b^2}\int_b^c T_2 r dr \quad (5)$$

Due to fluid pressure p_a,

$$u_{pf1} = \frac{1-\upsilon_1}{E_1}\frac{p_a a^2 - p_{\text{int} f} b^2}{b^2-a^2} r + \frac{1+\upsilon_1}{E_1}\frac{a^2 b^2}{r}\frac{p_a - p_{\text{int} f}}{b^2-a^2},$$

$$w_{pf1} = \frac{2\upsilon_1 L}{E_1}\frac{p_a a^2 - p_{\text{int} f} b^2}{b^2-a^2} \quad (6)$$

$$u_{pf2} = \frac{1-\upsilon_2}{E_2}\frac{p_{\text{int} f} b^2 - p_c c^2}{c^2-b^2} r + \frac{1+\upsilon_2}{E_2}\frac{b^2 c^2}{r}\frac{p_{\text{int} f} - p_c}{c^2-b^2},$$

$$w_{pf2} = \frac{2\upsilon_2 L}{E_2}\frac{p_{\text{int}f}b^2 - p_c c^2}{c^2 - b^2} \quad (7)$$

Where, p_{intf} is the interfacial radial pressure developed due to fluid pressure.

To find $p_{int\,f}$, at the interface (r = b), $u_{pf1} = u_{pf2}$
Due to interfacial radial straining pressure p (due to thermal mismatch properties),

$$u_{p1} = \frac{1-\upsilon_1}{E_1}\frac{pb^2}{b^2-a^2}r + \frac{1+\upsilon_1}{E_1}\frac{a^2b^2}{r}\frac{p}{b^2-a^2},$$
$$w_{p1} = \frac{2\upsilon_1 L}{E_1}\frac{pb^2}{b^2-a^2} \quad (8)$$

$$u_{p2} = \frac{1-\upsilon_2}{E_2}\frac{-pb^2}{c^2-b^2}r + \frac{1+\upsilon_2}{E_2}\frac{b^2c^2}{r}\frac{-p}{c^2-b^2},$$
$$w_{p2} = \frac{2\upsilon_2 L}{E_2}\frac{-pb^2}{c^2-b^2} \quad (9)$$

Due to interfacial axial straining stresses (σ_{zm1} and σ_{zm2}),

$$u_{a1(r=b)} = \frac{\upsilon_1}{E_1}\sigma_{zm1}b, \; w_{a1(r=b)} = \frac{\sigma_{zm1}}{E_1}L \quad (10)$$

For compatibility, the interfacial axial force F,

$$\sigma_{zm1}A_1 = -\sigma_{zm2}A_2 \quad (11)$$

$$u_{a2(r=b)} = \frac{\upsilon_2}{E_2}\frac{\sigma_{zm1}A_1}{A_2}b, \; w_{a2(r=b)} = -\frac{\sigma_{zm1}A_1}{E_2A_2}L \quad (12)$$

To find the interfacial straining pressure p and interfacial straining axial stress at (r = b)

$$u_{t1\,(r=b)} + u_{pf1\,(r=b)} + u_{p1\,(r=b)} + u_{a1\,(r=b)} = u_{t2\,(r=b)} + u_{pf2\,(r=b)} + u_{p2\,(r=b)} + u_{a2\,(r=b)} \quad (13)$$

$$w_{t1\,(r=b)} + w_{pf1\,(r=b)} + w_{p1\,(r=b)} + w_{a1\,(r=b)} = w_{t2\,(r=b)} + w_{pf2\,(r=b)} + w_{p2\,(r=b)} + w_{a2\,(r=b)} \quad (14)$$

Now the total principal stresses in layer 1 and layer 2 are: (i refers to 'i'th layer).

$$\sigma_{ri} = \sigma_{rti} + \sigma_{rpfi} + \sigma_{rpi}; \quad \sigma_{\theta i} = \sigma_{\theta ti} + \sigma_{\theta pfi} + \sigma_{\theta pi};$$
$$\sigma_{zi} = \sigma_{zti} + \sigma_{zmi} \quad (15)$$

2.3 Stresses due to fluid pressure

The compound cylinder is subjected to internal fluid pressure with the boundary conditions at r = a, pressure p = p_a and at r = c, pressure p = p_c. Layer 1 and layer 2 are taken separately in the evaluation of stresses. At r = b, i.e. p = $p_{intf,}$ interfacial pressure is compressive on layer 1 and tensile on layer 2 due to fluid pressure. Applying these boundary conditions the stresses are:

$$\sigma_{rpf1} = \frac{p_a a^2 - p_{\text{int}f}b^2}{b^2-a^2} - \frac{a^2b^2}{r^2}\frac{p_a - p_{\text{int}f}}{b^2-a^2}$$
$$\sigma_{\theta pf1} = \frac{p_a a^2 - p_{\text{int}f}b^2}{b^2-a^2} + \frac{a^2b^2}{r^2}\frac{p_a - p_{\text{int}f}}{b^2-a^2} \quad (16)$$
$$u_{rpf1} = \frac{1-\upsilon_1}{E_1}\frac{p_a a^2 - p_{\text{int}f}b^2}{b^2-a^2}r + \frac{1+\upsilon_1}{E_1}\frac{a^2b^2}{r}\frac{p_a - p_{\text{int}f}}{b^2-a^2}$$

$$\sigma_{rpf2} = \frac{p_{\text{int}f}b^2 - p_c c^2}{c^2-b^2} - \frac{b^2c^2}{r^2}\frac{p_{\text{int}f} - p_c}{c^2-b^2},$$
$$\sigma_{\theta pf2} = \frac{p_{\text{int}f}b^2 - p_c c^2}{c^2-b^2} + \frac{b^2c^2}{r^2}\frac{p_{\text{int}f} - p_c}{c^2-b^2}, \quad (17)$$
$$u_{rpf2} = \frac{1-\upsilon_2}{E_2}\frac{p_{\text{int}f}b^2 - p_c c^2}{c^2-b^2}r + \frac{1+\upsilon_2}{E_2}\frac{b^2c^2}{r}\frac{p_{\text{int}f} - p_c}{c^2-b^2}$$

2.4 Stresses due to interfacial mechanical pressure

Due to mismatch in thermal and mechanical properties of the two layers there is an interfacial mechanical pressure p at the interface r = b.

$$\sigma_{rp1} = \frac{pb^2}{b^2-a^2} - \frac{a^2b^2}{r^2}\frac{p}{b^2-a^2},$$
$$\sigma_{\theta p1} = \frac{pb^2}{b^2-a^2} + \frac{a^2b^2}{r^2}\frac{p}{b^2-a^2}, \quad (18)$$
$$u_{rp1} = \frac{1-\upsilon_1}{E_1}\frac{pb^2}{b^2-a^2}r + \frac{1+\upsilon_1}{E_1}\frac{a^2b^2}{r}\frac{p}{b^2-a^2}$$

$$\sigma_{rp2} = \frac{-pb^2}{c^2-b^2} - \frac{b^2c^2}{r^2}\frac{-p}{c^2-b^2},$$
$$\sigma_{\theta p2} = \frac{-pb^2}{b^2-a^2} - \frac{b^2c^2}{r^2}\frac{-p}{b^2-a^2}, \quad (19)$$
$$u_{rp2} = \frac{1-\upsilon_2}{E_2}\frac{-pb^2}{c^2-b^2}r + \frac{1+\upsilon_2}{E_2}\frac{b^2c^2}{r}\frac{-p}{c^2-b^2}$$

3. Mathematical Model Development for Multi-layer Concept of FGM

The functionally graded ceramic-metal material with the thermo-mechanical properties varying linearly along the radial direction, as an effect of the smooth variation of the volume fractions of the constituents, is assumed to be composed of multiple layers of very small elementary thickness in the radial direction as shown in Fig. 2.

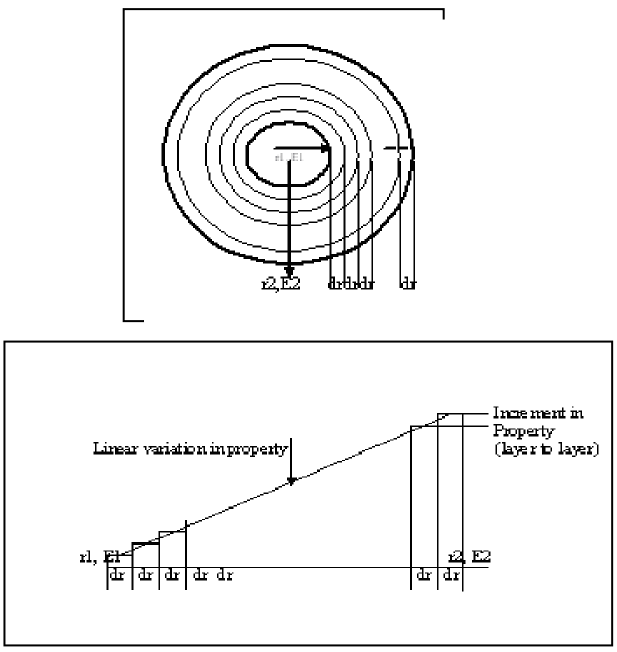

Fig. 2 FGM Split-up into Multi-layer Concept

The functionally graded circular hollow cylinder with inner radius '*a*' and outer radius c is split up into 'N' fictitious layers of equal radial thickness 'dr'.

The equations obtained for the two layer compound cylinder is extended to the multi-layer concept of the functionally graded circular hollow cylinder by taking two consecutive layers in succession and applying the boundary conditions at the surfaces and the compatibility conditions at each interface.

3.1 Stress-temperature relations

Applying the condition ($r = b = r_i$) at the interface into the equations (1) to (3) the stress-temperature relations turns out to be

$$\sigma_{ri} = \frac{E_{1i}\alpha_{1i}}{1-\upsilon_{1i}}\frac{1}{r^2}\left\{\frac{r^2-a^2}{r_i^2-a^2}\int_a^{r_i} T_r r dr - \int_a^r T_r r dr\right\}$$

$$\sigma_{\theta i} = \frac{E_{1i}\alpha_{1i}}{1-\upsilon_{1i}}\frac{1}{r^2}\left\{\frac{r^2+a^2}{r_i^2-a^2}\int_a^{r_i} T_r r dr + \int_a^r T_r r dr - T_r r^2\right\} \quad (20)$$

$$\sigma_{zi} = \frac{E_{1i}\alpha_{1i}}{1-\upsilon_{1i}} - \left\{\frac{2}{r_i^2-a^2}\int_a^{r_i} T_r r dr - T_r\right\}$$

where i (= 1 to N) refers to the 'i[th] layer. 1i refers to layer 1 to i[th] layer. Each layer is split into sub-layers of radius r, and using the equations (20) the stress at any layer is taken to be the average of the stresses at sub-layer radial positions r.

3.2 Radial and axial displacements

Applying the condition ($r = b = r_i$) at the interface of the 'i' th layer into the equations (4) to (14) the radial and axial displacements:
Due to temperature gradient are

$$u_{t1i} = r_i\,\alpha_{1i}\,T_i,\quad w_{t1i} = L\,\alpha_{1i}\,Ti \quad (21)$$

$$u_{tiN} = r_i\,\alpha_{iN}\,T_i,\quad w_{tiN} = L\,\alpha_{tiN}\,Ti \quad (22)$$

where, 1i refers to (for layer 1 to i[th] layer) and iN refers to (for layer i to N-th layer).

Due to fluid pressure p_a,

$$u_{pf1i} = \frac{1-\upsilon_{1i}}{E_{1i}}\frac{p_a a^2 - p_{\text{int}f} r_i^2}{r_i^2-a^2} r + \frac{1+\upsilon_{1i}}{E_{1i}}\frac{a^2 r_i^2}{r}\frac{p_a - p_{\text{int}f}}{r_i^2-a^2},$$

$$w_{pf1i} = \frac{2\upsilon_{1i}L}{E_{1i}}\frac{p_a a^2 - p_{\text{int}f} r_i^2}{r_i^2-a^2} \quad (23)$$

$$u_{pfiN} = \frac{1-\upsilon_{iN}}{E_{iN}}\frac{p_{\text{int}f} r_i^2 - p_c c^2}{c^2-r_i^2} r + \frac{1+\upsilon_{iN}}{E_{iN}}\frac{r_i^2 c^2}{r}\frac{p_{\text{int}f} - p_c}{c^2-r_i^2},$$

$$w_{pfiN} = \frac{2\upsilon_{iN}L}{E_{iN}}\frac{p_{\text{int}f} r_i^2 - p_c c^2}{c^2-r_i^2} \quad (24)$$

Where, p_{intf} is the interfacial radial pressure developed due to fluid pressure at each interface.

To find $p_{\text{int}f}$, at the interface ($r = b = r_i$), $u_{pf1i} = u_{pfiN}$
Due to interfacial radial straining pressure p (due to thermal mismatch properties),

$$u_{p1i} = \frac{1-\upsilon_{1i}}{E_{1i}}\frac{p r_i^2}{r_i^2-a^2} r + \frac{1+\upsilon_{1i}}{E_{1i}}\frac{a^2 r_i^2}{r}\frac{p}{r_i^2-a^2},$$

$$w_{p1i} = \frac{2\upsilon_{1i}L}{E_{1i}}\frac{p r_i^2}{r_i^2-a^2} \quad (25)$$

$$u_{piN} = \frac{1-\upsilon_{iN}}{E_{iN}}\frac{-p r_i^2}{c^2-r_i^2} r + \frac{1+\upsilon_{iN}}{E_{iN}}\frac{r_i^2 c^2}{r}\frac{-p}{c^2-r_i^2},$$

$$w_{piN} = \frac{2\upsilon_{iN}L}{E_{iN}}\frac{-p r_i^2}{c^2-r_i^2} \quad (26)$$

Due to interfacial axial straining stresses (σ_{zm1i} and σ_{zmiN}),

$$u_{a1i(r=b)} = \frac{\upsilon_{1i}}{E_{1i}}\sigma_{zm1i} r_i,\quad w_{a1i(r=b)} = \frac{\sigma_{zm1i}}{E_{1i}} L \quad (27)$$

For compatibility, the interfacial axial force F,

$$\sigma_{zm1i}A_{1i} = -\,\sigma_{zmiN}A_{iN} \quad (28)$$

$$u_{aiN(r=b)} = \frac{\upsilon_{iN}}{E_{iN}} \frac{\sigma_{zm1i} A_{1i}}{A_{iN}} r_i \,,\; w_{aiN(r=b)} = -\frac{\sigma_{zm1i} A_{1i}}{E_{iN} A_{iN}} L \quad (29)$$

To find the interfacial straining pressure p and interfacial straining axial stress at (r = b), total radial and axial displacements,

$$u = u_{t1i} + u_{pf1i} + u_{p1i} + u_{a1i} = u_{tiN} + u_{pfiN} + u_{piN} + u_{aiN} \quad (30)$$

$$w = w_{t1i} + w_{pf1i} + w_{p1i} + w_{a1i} = w_{tiN} + w_{pfiN} + w_{piN} + w_{aiN} \quad (31)$$

Now the total principal stresses in layer i are: (i refers to 'i'th layer).

$$\sigma_{ri} = \sigma_{rti} + \sigma_{rpfi} + \sigma_{rpi}, \quad \sigma_{\theta i} = \sigma_{\theta ti} + \sigma_{\theta pfi} + \sigma_{\theta pi}, \quad \sigma_{zi} = \sigma_{zti} + \sigma_{zmi} \quad (32)$$

3.3 Stresses due to fluid pressure

Applying the condition (r = b = r_i) at the interface into the equations (16) and (17)

$$\sigma_{rpfi} = -p_{intfi},$$

$$\sigma_{\theta pfi} = \frac{-p_{intfi}\left(r_i^2 + r_{i-1}^2\right) + 2\, p_{intf(i-1)}\, r_{i-1}^2}{\left(r_i^2 - r_{i-1}^2\right)} \quad (33)$$

Where suffices 'i' and 'i-1' refer to the i^{th} layer and $(i-1)^{th}$ layers respectively.

3.4 Stresses due to interfacial mechanical pressure

Applying the condition (r = b = r_i) at the interface into the equations (18) and (19)

$$\sigma_{rpi} = -p_i,$$

$$\sigma_{\theta pi} = \frac{-p_i\left(r_i^2 + r_{i-1}^2\right) + 2\, p_{i-1}\, r_{i-1}^2}{\left(r_i^2 - r_{i-1}^2\right)} \quad (34)$$

4. Solutions

The functionally graded circular hollow cylinder is assumed to be subjected to steady-state temperature loads T_{in} at inner surface (r = *a*), T_{out} at outer surface (r = *c*). The cylinder is also subjected to internal pressure p_a (r = *a*) and external pressure p_c(r = c) or atmospheric pressure.

4.1 Temperature distributions

The thermal load is specified as heat flux, q' at inner surface. In this case first the temperature at each interface of the N layers is obtained as:

Case 1: heat flux, q' specified at r = a and temperature fixed at 0°C at r = c.

$$T_i = \frac{q'\,a}{k_{iN}} \ln \frac{c}{r_i} \quad (35)$$

where k_{iN} refer to mean thermal conductivity for the layers i to N.

4.2 Radial and axial displacements

Due to temperature gradient the displacements are obtained from equations (21) and (22). Interfacial fluid pressure, p_{intf} is obtained from ($u_{pf1i} = u_{pfiN}$) and then the displacements due to interfacial fluid pressure are obtained from the equations (23) and (24). Interfacial straining pressure, p and interfacial axial straining stress σ_{zm1i} are obtained from equations (30) and (31) equating the total radial and axial displacements at 'i' th interface for layers 1 to i with that for the layers i to N. Substituting the value of p into equations (25) and (26) gives the displacements due to interfacial straining pressure, p. Substituting the value of σ_{zm1i} into equations (27) and (29) gives the displacements due to interfacial axial straining stress, σ_{zm1i}. The total radial and axial displacements for the 'i'th layer, u_i and w_i are obtained from equations (30) and (31).

4.3 Radial, hoop and axial stresses

Stresses in the radial, hoop and axial directions due to temperature differences is given by equations (20), due to fluid pressure by equations (33) and due to interfacial mechanical pressure by equations (34). The total stresses in the three directions are given by equations (32).

5. Conclusion

The mathematical model to analyse cylindrical shell with coating (two layered isotropic cylindrical component) subjected with internal fluid pressure and thermo-mechanical load has been described.

The methodology has been established to analyse the functionally graded circular hollow cylinder on the assumed equivalent isotropic multi-layers under thermo-mechanical and fluid pressure. The governing equations have been obtained and the solutions for the temperature, stresses and displacements have been established.

The mathematical model can be used in functionally graded materials which play an important role in the applications of cylindrical components subjected to higher temperature gradients.

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