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SQUEEZE FILM CHARACTERISTICS OF COUPLE STRESS FLUID BETWEEN POROUS TRIANGULAR PLATES

V.Sudha[1], Sundarammal Kesavan[2], V.Ramamurthy[3]
[1]Lecturer & Research Scholar, Department of Mathematics,
[2]Professor, Department of Mathematics,
[3]Professor & Head, Department of Mathematics,
SRM University, Kattankulathur, Tamil Nadu, India.
Email: sudhaveer@yahoo.co.in

ABSTRACT

A theoretical study of hydrodynamic squeeze film behaviour for triangular plates is analysed. The upper nonporous triangular plate is approaching the homogenous and isotropic lower porous triangular plate. The lubricant between the plates is a couple stress fluid with no body forces and body couples. On the basis of microcontinuum, theory, the Stokes constitutive equations are solved and the modified Reynolds equation is derived. It is assumed that the polymer additives present in the lubricant do not percolate in the lower porous plate ie., it is assumed that the size of the polar additives present in the couple stress is much greater than the pore size of the lower plate. An expression for squeeze film pressure is obtained. The load carrying capacity of the squeeze film is obtained by using the squeeze film pressure. The bearing characteristics of the couple stress lubricant are then obtained and the results are compared to the Newtonian case. The results reveal that the presence of micro structures in the fluid film cause an enhancement of squeeze film characteristics showing that couple stress lubricants are better fluids than Newtonian fluids.

Keywords: Squeeze film, Couple stress fluid, permeability.

1. INTRODUCTION

Lubrication is the art of reducing frictional resistance by means of the introduction of a foreign substance introduced between two surfaces having relative motion. Machines like jet engines, rolling mills, wrist watches, grinding wheel spindles, electric motors, auto engines, electric generating equipments use some kind of bearing which is found as the vital and indispensable element. Physical properties such as density, viscosity, heat capacity, thermal conductivity and the temperature – pressure viscosity etc., determine the ability of the lubricant to operate under hydrodynamic lubrication. A wide variety of theories were discussed by several authors in the analysis of laminar squeeze flow of incompressible viscous fluids. Squeeze films of some Non-Newtonian fluids like power law fluids, micro polar fluids, fluids with couple stress have been investigated in [1,2]The addition of small additives to a Newtonian fluid provides beneficial effects on the load – carrying and frictional characteristics (Oliver) [3].

The Stokes [4] is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of couple stresses, body couples and non – symmetric tensors. The squeeze film characteristics on thrust bearings lubricated by couple stress have been analysed in [5]. Bujurke and Jeyaraman [6] have analyzed the influence of couple stress fluid in a squeeze film configuration with reference to synovial joints. Jaw Ren Lin [7] predicted the presence of couple stress provides an enhancement in the load carrying capacity, and lengthens the response time of the squeeze film in long journal bearings.

Porous bearings have been widely used in industry for a long time. The aspects of porous metal bearings were studied by Kumer in 1980. The fluid flow in a composite region, which is partially filled with the fluid film, is of considerable interest. Most studies deal primarily with the mathematical formulation based on Darcy's Law, which neglects the effect of a solid boundary on fluid flow through porous media (Morgan and Cameron, 1957; Cusano 1972; Murti 1973) Beaver and Joseph, 1967 [8] experimentally showed that slip flow takes place over a permeable boundary. The slip flow boundary conditions were applied to porous bearing and were discussed by Murti . The velocity distribution in porous medium is no longer uniform and the distortion of velocity yields the viscous shearing stress when the porous layer is of shallow depth.

In [9] ,it was analysed that shear terms in the Brinkman's equations assures the existence of boundary layer within the porous medium. The static and dynamic properties of porous bearings were discussed by Lin and Hwang (1993, 1974). The results show that the effects of viscous shear will increase the low capacity and decrease the co-efficient of friction.

In [11] the squeeze film characteristics have been analysed for anisotropic porous rectangular plates by taking into account the slip velocity at the fluid and porous material interface. In [12] the squeeze film

Corresponding author: *sudhaveer@yahoo.co.in*

lubrication between two isotropic porous rectangular plates has been analysed. In our previous studies [13,14,15,16] we have investigated the effects of some Non-Newtonian fluids between rectangular, elliptic and triangular plates The purpose of this paper is to analyze the squeeze film characteristics of porous triangular plates with a couple stress lubricant. The film pressure is solved using the modified Reynolds equation by taking into account the couple stress, isotropic permeability and velocity slip parameters. The squeeze time obtained from the film pressure for a couple stress fluids using Numerical Integration and the results are compared to the Newtonian case.

2 FORMULATION AND SOLUTION OF THE PROBLEM

The squeezing of a viscous fluid between parallel triangular plates is considered. The upper plate is approaching a lower porous plate. The lower plate is stationary and the upper plate moves slowly towards the lower plate. The fluid between the plates and in the porous matrix is assumed to be a Stokes couple stress fluid. It is assumed that the fluid is incompressible with no body forces or couples.

The Stokes equations of motion for a couple stress fluid in the film region is

$$\nabla.\bar{q} = 0 \qquad (1)$$

$$\rho\frac{D\bar{q}}{Dt} = -\nabla p + \mu\nabla^2\bar{q} - \eta\nabla\bar{q} \qquad (2)$$

where,

$\bar{q}$ = (u, v, w) is the velocity, ρ is the density, p is the pressure, μ is the material constant with dimension of viscosity and η is the material constant with dimension of momentum.

From the usual assumptions of hydrodynamic lubrications applicable to thin films, the equations of motions in the film region are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3)$$

$$\mu\frac{\partial^2 u}{\partial z^2} - \eta\frac{\partial^4 u}{\partial z^2} = \frac{\partial p}{\partial x} \qquad (4)$$

$$\mu\frac{\partial^2 v}{\partial z^2} - \eta\frac{\partial^4 v}{\partial z^2} = \frac{\partial p}{\partial y} \qquad (5)$$

$$\frac{\partial p}{\partial z} = 0 \qquad (6)$$

The flow of couple stress fluid in the porous matrix is governed by the modified form of the Darcy's law, which accounts for the polar effect is

$$q^* = \frac{-k}{\mu(1-\beta)}\nabla p^* \qquad (7)$$

where q* = (u*, v*, w*), u*, v*, w* are the Darcy velocity components along x, y, z directions.

Here p* is the pressure in the porous region and k is the isotropic permeability of the porous matrix and $\beta = \frac{\eta}{\mu k}$ represents the ratio of the microstructure size to the pore size.

The pressure in the porous medium satisfies the Laplace Equation

$$\nabla^2 p^* = 0 \qquad (8)$$

The boundary conditions for the velocity components at z = h are

$$u = v = 0 \qquad (9)$$

$$w = -\frac{\partial h}{\partial t} \qquad (10)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0 \qquad (11)$$

where h is the film thickness.

The boundary conditions for the velocity components at z = 0 are

$$\frac{\alpha}{\sqrt{k}}\left(u - u^*\right) = \frac{\partial u}{\partial y} \qquad (12)$$

$$\frac{\alpha}{\sqrt{k}}\left(v - v^*\right) = \frac{\partial v}{\partial z} \qquad (13)$$

$$w = -w^* \qquad (14)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0 \qquad (15)$$

Equations (12) & (13) are the Beavers and Joseph [8] slip boundary conditions for the tangential velocity slip at the porous interface. Here α is the slip co-efficient which is a dimensionless quantity depending on the material parameters and characterizes the structure of the permeable material within the boundary region.

The solutions of equations (3) & (5) satisfying the above boundary conditions as obtained by Naduvinamani [12] is

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{12\mu}{j(h,c,b,l)}\left[\frac{\partial h}{\partial t}+\frac{k}{\mu(1-\beta)}\left.\frac{\partial p^*}{\partial z}\right|_{z=0}\right] \quad (16)$$

where $j(h,c,b,l)=$

$$h^3(1+b)-6h^2lc\tanh\left(\frac{h}{2l}\right)-12l^2\left(h-2l\tanh\left(\frac{h}{2l}\right)\right) \quad (17)$$

$$c=\frac{s}{(s+\bar{h})}, \quad (18)$$

$$b=\frac{3s}{(1-\beta)}\left[\frac{2s\alpha^2}{\left(\bar{h}^2+s\bar{h}\right)}+\frac{(1-\beta)}{(\bar{h}+s)}\right] \quad (19)$$

and

$$\bar{h}=\frac{h}{h_O}, s=\frac{\sigma}{h_O}, \sigma=\frac{\sqrt{k}}{\alpha}, l=\sqrt{\frac{\eta}{\mu}} \quad (20)$$

The relevant boundary conditions for the pressure are

p = p* = 0 on the boundary of the plate (21)

$$\frac{\partial p^*}{\partial z}=0 \quad \text{at } z = -\delta \quad (22)$$

$p = p^*$ at $z = 0$ (23)

where δ is the thickness of the porous layer and is very small.

Solving (8) using (22)

$$\left(\frac{\partial p^*}{\partial z}\right)_{z=0}=-\int_{-\delta}^{0}\left(\frac{\partial^2 p^*}{\partial x^2}+\frac{\partial^2 p^*}{\partial y^2}\right)dz$$

i.e $$\left(\frac{\partial p^*}{\partial z}\right)_{z=0}=-\delta\left(\frac{\partial^2 p^*}{\partial x^2}+\frac{\partial^2 p^*}{\partial y^2}\right) \quad (24)$$

Since at z = 0, p = p*, Eq (24) reduces to

$$\left(\frac{\partial p^*}{\partial z}\right)_{z=0}=-\delta\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right) \quad (25)$$

Substituting (25) in equation (16)

$p_{xx} + p_{yy} = R$ (26)

where $$R=\left(\frac{12\mu\dot{h}}{f+12\delta k/(1-\beta)}\right), \quad (27)$$

Assume the equations of the boundary of the plates as $y = \pm\frac{x}{\sqrt{3}}$ and $x = \frac{\sqrt{3}}{2}a$ (i.e) the triangular plate is equilateral. Then

$$p=\frac{-R}{2a\sqrt{3}}(x^2-3y^2)\left(\frac{\sqrt{3}a}{2}-x\right) \quad (28)$$

satisfies equation (26) with p = 0 on the boundary of the plates.

The load carrying capacity W of the squeeze film is obtained by integrating the pressure field over the surface of top plate.

$$W=\iint_A p(x,y)\,dx\,dy \quad (29)$$

where A is the area of the upper plate.

Substituting (28) in (29)

$$W=\frac{-Ra^4}{960\sqrt{3}} \quad (30)$$

The non –dimensional load-carrying capacity is

$$\overline{W}=\frac{-Wh^3}{\mu A^2\dot{h}} \quad (31)$$

$$\overline{W}=\frac{Ra^4h^3}{\mu A^2\dot{h}960\sqrt{3}} \quad (32)$$

$$\overline{W}=\frac{1}{15\sqrt{3}}\left(\frac{1}{J(\bar{h},c,b,\bar{l})+12\psi/(\bar{h}^3(1-\beta)}\right) \quad (33)$$

where $=J(\bar{h},c,b,\bar{l})=\frac{j(h,c,b,l)}{h_3}$

$$=(1+b)-6c\frac{\bar{l}}{\bar{h}}\tanh\left(\frac{\bar{h}}{2\bar{l}}\right)-12\frac{\bar{l}^2}{\bar{h}^2}\left(1-2\frac{\bar{l}}{\bar{h}}\tanh\left(\frac{\bar{h}}{2\bar{l}}\right)\right) \quad (34)$$

$$\psi=\frac{\delta k}{h_0^3}, \bar{h}=\frac{h}{h_0} \text{ and } \bar{l}=\frac{\bar{l}}{h_0} \quad (35)$$

The dimensionless time is

$$T=\frac{16\sqrt{3}}{45}\left[\frac{-h_0^2Wt}{\mu a^4}\right] \quad (36)$$

$$T = \int_{1}^{\bar{h}} \left(\frac{1}{J(\bar{h}, c, b, \bar{l})\bar{h}^{3} + 12\psi/(1-\beta)} \right) d\bar{h} \quad (37)$$

3. RESULTS AND DISCUSSIONS

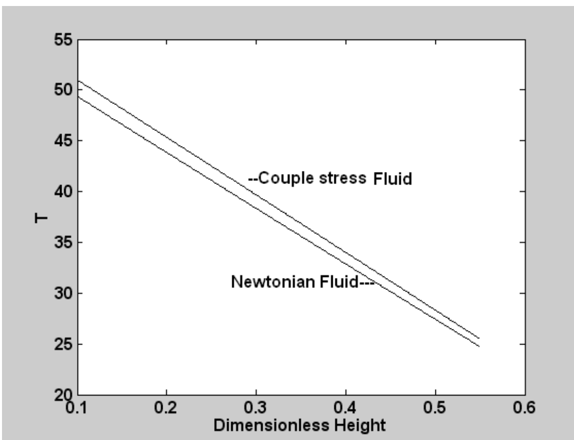

Fig 1 Variations in the dimensionless time with dimensionless height for couple stress fluid (with $\bar{l}$=0.2) and Newtonian fluid for s=0.25 and β=0.2.

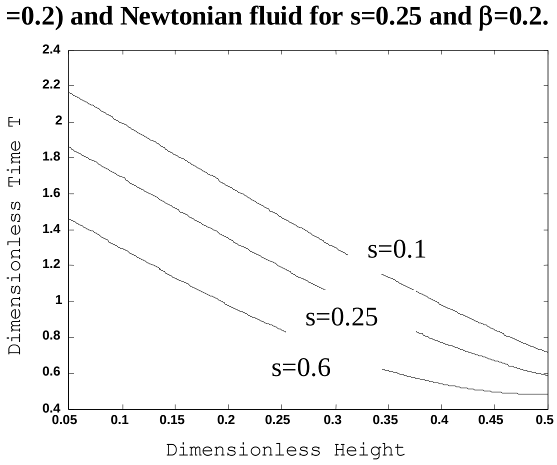

Fig. 2 Variations in the dimensionless time T for different values of s with $\bar{l}$=0.12, β=0.2 and ψ = 0.01.

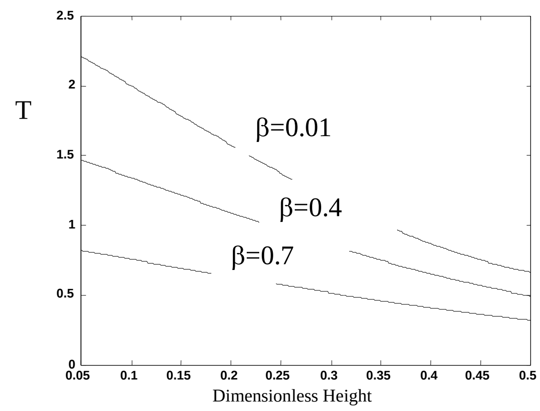

Fig. 3 Variations in the dimensionless time T for different values of β with $\bar{l}$=0.12, s=0.25 and ψ = 0.02.

Equation (37) is solved using numerical integration and the dimensionless time T with $\bar{h}$ is obtained for the couple stress parameter $\bar{l}$ =0.2, s = 0.25 and β = 0.2

The dimensionless time T with $\bar{h}$ for a Newtonian fluid is obtained by taking $\bar{l}$ = 0 and β = 0.

Fig 1 shows the dimensionless time T is more for a couple stress fluid than the Newtonian fluid.

The effect of the slip parameter is studied for $\bar{l}$ = 0.12, β = 0.2 and ψ= 0.01

Fig 2 shows that an increase in the permeability parameter s decreases the dimensionless time T (i.e.) the presence of the tangential velocity slip at the porous interface reduces T.

The dimensionless time T with $\bar{h}$ is studied by varying the percolation parameter β for s = 0.25, $\bar{l}$ = 0.12 and ψ = 0.02.

Fig 3 shows that an increase in β decreases the dimensionless time T.

4. CONCLUSIONS

From the above discussions it is clear that the micro additives present in the lubricant enhances the squeeze film characteristics. This shows that couple stress fluids are better lubricants than Newtonian fluids.

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NOTATION

A	Area of the plate (m^2)
h	fluid-film thickness (m)
$\overline{h}$	non-dimensional film height $=h/h_0$
h_0	initial film thickness (m)
l	couple-stress parameter$=\sqrt{(\eta/\mu)}$ (m)
$\bar{l}$	non-dimensional couple-stress parameter $= l/h_0$
p	pressure in the film region (Pa)
p*	pressure in porous region (Pa)
s	slip parameter $= \sqrt{k}/\alpha h_0$
t	time (s)
T	non-dimensional squeeze-film time as defined in equation (36)
u,v,w	components of fluid velocity in film region (m/s)
u*,v*,w*	components of fluid velocity in porous region (m/s)
W	Load (N)
W	dimensionless load as defined in equation (31)
x,y,z	coordinates (m)
α	slip constant
β	percolation parameter = $(\eta/\mu)/k$
δ	thickness of the porous layer (m)
μ	isotropic viscosity (Pa s)
η	lubricant couple stress constant (N s)
ρ	density (kg/m^3)
ψ	permeability parameter = $k\delta/h_0^3$