PERFORMANCE EVALUATION OF A MULTI PRODUCT PRODUCTION SYSTEM USING TIMED EVENT GRAPH AND MAX- PLUS ALGEBRA

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ABSTRACT

The production systems used in the modern industry are usually composed of integrated and complex processing equipment having automated material handling and advanced computer networks for information transfer. Mathematical models are essential for understanding various events which are occurring in production systems. In this paper, max-plus algebra have been applied to model a complex production system known as multi product production system (MPPS) with different routing policies and its usefulness to describe and analyze such systems has been presented. An algorithm known as MPA-algorithm is proposed for analytical model building in max-plus algebra from the timed event graph (TEG) model. The basic criterion used for performance evaluation of the modeled system is cycle time, which is evaluated by using Karp's theorem from graph theory. Other performance measures of the MPPS are also evaluated from the max-plus model by treating MPPS as a flow-shop system.

Keywords: Timed Event Graph, Max-Plus Algebra, Karp's Theorem

1. INTRODUCTION

The max-plus algebra, which has maximization and addition as its basic operations, is one of the frameworks that can be used to model a class of discrete event dynamic systems (DEDS). A DEDS is dynamic, asynchronous system, where the state transitions are initiated by events that occur at discrete instants of time. Typical examples of DEDS are flexible manufacturing systems, telecommunication networks, traffic control systems… A class of mathematical modeling tool that can also be used for DEDS are Petri nets (PN). Petri nets give a graphical representation for DEDS, which closely resembles the physical system. While reviewing literature related to modeling of production systems, it is observed that Petri nets are extensively used for modeling and performance evaluation [1] [2] [3]. The technique usually adopted for analysis of Petri nets is either reachability matrix method or invariant approach. The problem with these techniques is that, the final matrix size significantly increases as the production system size in terms of processing units is increased, thus making the analysis difficult [3][4]. This is because the final matrix size of the model depends on number of places and transitions used in the PN model of the system rather than the number of events present in the system. To overcome this problem a new mathematical modeling tool based on max-plus algebra is used for production systems in the present work. Max-plus algebra is used in variety of applications namely manufacturing systems [5] [6], transportation systems [7], communication systems [8]. However a systematic modeling methodology in max-plus algebra is not reported in the literature.

The main objective of this paper is to gain insight into the performance evaluation of multi product production systems with different routing policies and to demonstrate the usefulness of max-plus algebra to efficiently analyze the problems of real time control of production systems. Performance measures in terms of cycle time of the system, average workload of machine, average cycle time, average flow time and average processing time of part are evaluated.

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2. BASICS OF PETRI NETS AND MAX-PLUS ALGEBRA

The structure of a Petri net [9] [10] is a bipartite directed graph consisting of two types of nodes called *places* and *transitions*. Places and transitions are joined by *directed arcs. Input* (respectively, *output*) places of a transition are places connected by incoming (respectively, outgoing) arcs of the transition. Formally, a Petri net is specified as a 4-tuple $N = (P, T, F, M_o)$ where:

$P = \{p_1, p_2, p_3 \dots\dots\dots, p_r\}$ is the finite set of places

$T = \{t_1, t_2, t_3 \dots\dots\dots, t_r\}$ is the finite set of transitions

$F \subseteq (P \times T) \cup (T \times P)$ is the set of directed arcs and

$M_o = P \rightarrow \{0, 1, 2 \dots\}$ is the initial marking of graph.

A transition t is *enabled* by a marking M_o if and only if each of its input places is marked with at least one token. An enabled transition may be chosen to fire. The *firing* of a transition consists of removing one token from each of its input places and adding one token to each of its output places (based on the cardinality value of arcs). In order to comprehend the process of modeling production systems, timed event graphs are used. A timed event graph is a subclass of Petri nets, where each place has only one input and output transition [10]. The aim of using TEG is to describe the behavior of the system by mathematical linear equations in max-plus algebra.

Max-plus algebra is normally defined as the system ($\underline{\Re}, \oplus, \otimes$), where $\underline{\Re} = \Re \cup \{\varepsilon\}$ is the set of real numbers with $\varepsilon = -\infty$ adjoined, and the symbols $\oplus$ and $\otimes$ present binary operations determined for any $a, b \in \underline{\Re}$ respectively as, $a \oplus b = \max(a, b)$, $a \otimes b = a + b$.

The neutral elements for the operators $\oplus$ and $\otimes$ are respectively $\varepsilon = -\infty$ and $e = 0$ ($\forall\, a \in \underline{\Re}$, $a \oplus \varepsilon = a$ and $a \otimes e = a$). Like other algebraic structures, the max-plus algebra have properties and characteristics such as the associativity of addition and the multiplication, the commutativity of addition, the distributivity of multiplication, existence of zero element (denoted ε). The scalar max-plus operations are extended to matrices in a standard way. Specifically, for any ($n \times n$) matrices X = (x_{ij}) and Y = (y_{ij}), the entries of U = X $\oplus$ Y and V = X $\otimes$ Y are calculated as

$u_{ij} = x_{ij} \oplus y_{ij}$, and $v_{ij} = \sum_{k=1}^{n} \oplus (x_{ik} \otimes y_{kj})$,

Where the symbol $\sum_{k=1}^{n} \oplus$ denotes the iterated operation $\oplus$. The matrix with all its elements equal to ε is the zero matrix, and the identity matrix E is defined as the diagonal matrix with all diagonal entries equal to e, i.e., $E_{ii} = e$ and $E_{ij} = \varepsilon$ for $j \neq i$.

Matrix powers in max-plus algebra have a special meaning in terms of paths in graph theory. By definition of matrix multiplication, $[A^2]_{ij} = \max_{k=1,\dots,n} (a_{ik} \oplus a_{kj})$, which is just the maximum weight of all paths from j to i with exactly two arcs. In general, A^l is the matrix of the maximum weights of paths with length l, and likewise $A^k = \bigoplus_{l=0}^{k} A^l$ is the matrix of the maximum weights of all paths with length equal to or smaller than k. Thus the longest or critical path matrix Ā is defined as

$$\bar{A} = \bigoplus_{l=1}^{\infty} A^l = A \oplus A^2 \oplus A^3 \oplus \dots .$$

The reader can refer [8] for more information on basics and applications of max-plus algebra. The next section explains the proposed MPA-algorithm for analytical model building in max-plus algebra.

3. MAX-PLUS ALGEBRA ALGORITHM (MPA-ALGORITHM)

The max-plus algebra formalism is useful for discrete event systems like production systems where cyclic nature of sequence of operations occurs. The input parameters required for this algorithm are: system parameters, which are the time of event executions on resources, an initial state, and sequence of resource usage which guarantee the cyclic system of operations. The steps involved in MPA-Algorithm are as follows:

1. Specify state vector x_i, input vector u_i and determine the elements of both the vectors

$$\left.\begin{aligned} x_i &= (x_i(1), x_i(2), \dots, x_i(k)), \\ u_i &= (u_i(1), u_i(2), \dots, u_i(k)) \end{aligned}\right\} \quad \text{----- (1)}$$

where, elements x_i (k) are the start time of operations (event executions) in the k^{th} iteration and elements u_i (k) are the time instant at which parts to be processed enter into the system for the k^{th} iteration.

2. Determine the elements of the matrices in linear state equations of max-plus algebra,

$$x(k) = A \otimes x(k-1) \oplus B \otimes u(k) \quad \text{---- (2)}$$
$$y(k) = C \otimes x(k) \quad \text{----(3)}$$

where, A is a matrix of size n × n (n is the size of the state vector), B is a matrix of size n × m (m is the number of input start state events) and C is a matrix of size p × n (p is the number of output start state events). The entries of A, B and C are in $\underline{\Re} = \Re \cup \{-\infty\} \cup \{+\infty\}$ and correspond to some delays or holding times associated to the places in TEG. For the system executing two events, say *i* and *j*,

the matrix A consists of the following elements:

(i) $A_{ij} = d_j$, if the j^{th} event precedes the i^{th} event and the j^{th} event *does not* precede directly to the beginning of a cycle;

(ii) $A_{ij} = \varepsilon$ in other cases, where d_j is the execution time of j^{th} operation.

the matrix B consists of the following elements:

(i) $B_{ij} = d_j$, if the j^{th} event precedes the i^{th} event and the j^{th} event *does not* precede directly to the beginning of a cycle;

(ii) $B_{ij} = e$, if the start of the i^{th} event is a required condition to start the j^{th} operation;

(iii) $B_{ij} = \varepsilon$ in other cases.

the matrix C consists of the following elements:

(i) $C_{ij} = d_j$, if the j^{th} event *precedes* the i^{th} event and the j^{th} event *does* precede directly to the beginning of a next cycle;

(ii) $C_{ij} = \varepsilon$ in other cases,

3. Introduce an identity matrix $E \in \underline{\Re}^{n \times n}$ to describe the dynamics of restarting for the next cycle.

$$u(k) = E \otimes y(k-1) \quad \text{---- (4)}$$

where, y(*k*-1) is the time instant at which (*k*-1)th event finishes and is the last event in the system and *u*(*k*) is the time instant at which k^{th} event starts.

4. Substitute equation (4) in equation (2)

$$x(k) = A \otimes x(k-1) \oplus B \otimes E \otimes y(k-1) \quad \text{--- (5)}$$

5. Modify the equation (3) for (k-1)th cycle and substitute *y*(*k*-1) in equation (5)

$$\begin{aligned} x(k) &= A \otimes x(k-1) \oplus B \otimes E \otimes C \otimes x(k-1) \\ &= (A \oplus B \otimes E \otimes C) \otimes x(k-1) \\ &= \hat{A} \otimes x(k-1) \quad \text{--- (6)} \end{aligned}$$

where: $\hat{A} = (A \oplus B \otimes E \otimes C)$

6. Use following max-plus expression for difference in the same state variables of two subsequent cycles to determine the value of cycle time λ.

$$x(k) = \lambda \otimes x(k-1) \quad \text{---- (7)}$$

7. Determine the value of cycle time λ from equation (6) and (7)

$$\hat{A} \otimes x(k-1) = \lambda \otimes x(k-1)$$

where λ is a max-plus eigen value of the matrix Â. The eigen value is determined by using Karp's theorem.

3.1 Karp's Theorem Statement

Given an n × n matrix A, with corresponding precedence graph Ĝ = (ν, δ), where ν is a set of elements called nodes and δ is a set the elements of which are ordered pairs of nodes called arcs. The maximum cycle mean λ is given by

$$\lambda = \max_{i=1\ldots n} \min_{k=0\ldots n-1} [(A^n)_{ij} - (A^k)_{ij}]/(n-k) \quad \forall j \quad \text{-- (8)}$$

In the equation (8), A^n and A^k are to be evaluated as in max-plus algebra; the other operations are conventional ones. The index *j* is arbitrary and one can take any *j* ∈ {1... *n*}. The resulting value of *λ* is independent of *j* [8].

4. DESCRIPTION OF A MULTI PRODUCT PRODUCTION SYSTEM

The multi product production system considered here as a case study consists of four machines and four parts. The system is to produce all four kinds of parts according to a certain product mix. The routes followed by each part and each machine are depicted in Fig.1 in which M_i, *i* = 1,2,3,4 are machines and P_i, *i* =1, 2, 3, 4 are the parts. Processing times of parts P_1, P_2, P_3 and P_4 on various machines are given in Table 1.

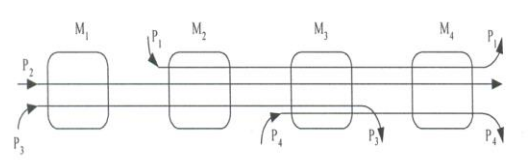

Fig 1. Routing of parts along machines in MPPS

Table 1: Processing times (d_i) of machines

Parts/ Machines	P_1	P_2	P_3	P_4
M_1	-	1	5	-
M_2	3	3	4	-
M_3	5	2	7	4
M_4	4	6	-	3

Parts of type P_1 first visit machine M_2 next goes to M_3 and then go to M_4. Parts of type P_2 enter the system via machine M_1, then they go to M_2 and then to M_3 finally leaving the system through machine M_4. Parts of type P_3 enter the system at M_1 next goes to M_2 and leaves the system through M_3. Parts of type P_4 first visit M_3 and then go to M_4. The characteristics of the MPPS are as follows:

- Parts are carried around on pallets and there is one pallet available for each type of part.
- Transportation times are negligible and there are no setup times on the machines when they switch from one part type to another.
- Sequencing of the various parts on the machines is known: on machine M_1 it is (P_2, P_3), i.e., the machine first processes a part of type P_2 and then part of type P_3, on machine M_2 it is (P_1, P_2, P_3), on machine M_3 it is (P_1, P_2, P_3,P_4) and (P_1, P_2, P_4) on machine M_4. These sequences are called local dispatching rules.
- The final product mix is balanced in the sense that it can be obtained by means of a periodic input of parts, here chosen to be P_1, P_2, P_3, P_4.

The information about the sequencing and the duration (processing times) of the various events is shown in Fig. 2 with the help of precedence graph.

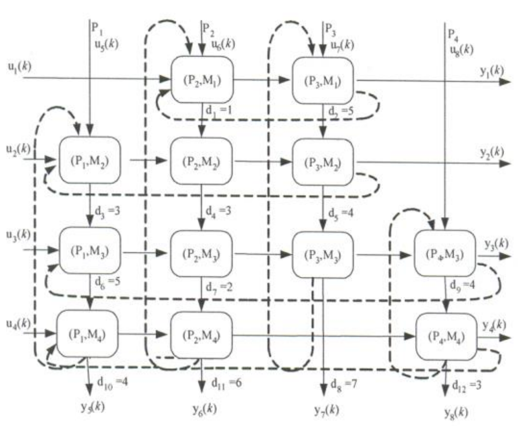

Fig 2. Precedence graph for MPPS with feedback arcs

In the Fig.2, the events are represented by ordered pairs of the form (P_i, M_j) meaning that part of type P_i is processed on machine M_j. The arcs represent the precedence constraint and the dotted line shows the feedback arc indicating the beginning of next sequence. It means that, after a machine has finished a sequence of products, it starts with the next sequence. If the pallet on which product P_i was mounted is at the end, the finished product is removed and the empty pallet immediately goes back to the starting point to pick up a new product P_i.

The timed event graph model of the MPPS is shown in Fig 3. This TEG model is constructed from the events identified in MPPS. Here the timing under consideration (i.e., duration of events) is limited to constant holding times on places. The firing times of all the transitions are assumed to be zero and they are called as immediate transitions. The start of identified events in the system is shown with transitions, which are given in Table 2. For example the event x_1 which describe 'M_1 is ready to process P_2' is shown with two places p_3 and p_4. Here p_3 indicates the processing of P_2 by machine M_1 and p_4 indicates the part P_2 is busy. Similarly the interpretations for other events are to be made.

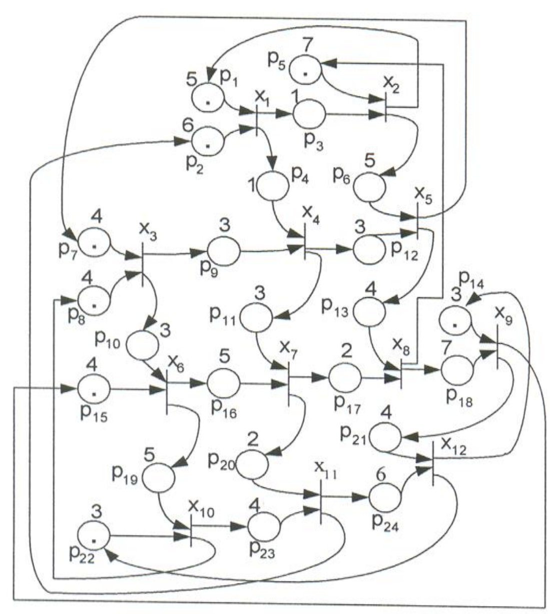

Fig 3. Timed event graph model of MPPS

Table 2: Description of events

Transition	Description
x_1	M_1 is ready to process P_2
x_2	M_1 is ready to process P_3
x_3	M_2 is ready to process P_1
x_4	M_2 is ready to process P_2
x_5	M_2 is ready to process P_3
x_6	M_3 is ready to process P_1
x_7	M_3 is ready to process P_2
x_8	M_3 is ready to process P_3
x_9	M_3 is ready to process P_4
x_{10}	M_4 is ready to process P_1
x_{11}	M_4 is ready to process P_2
x_{12}	M_4 is ready to process P_4

5. PERFORMANCE EVALUATION IN MAX-PLUS ALGEBRA

This section gives the evolution of max-plus algebra equations from the timed event graph model of the MPPS. Let

u_i (k) → Time instant at which machine M_i is available for the first activity i = 1,2,3,4.

u_j (k) → Time instant at which the raw material for a part of type P_{j-4} is available in the k^{th} production cycle for j = 5,6,7,8.

x_i (k) → Time instant at which activity i starts in the k^{th} production cycle for i = 1, 2,…,12.

y_i (k) → Time instant at which machine M_i has finished processing the last part of k^{th} processing cycle i=1,2, 3, 4

y_j (k) → Time instant at which finished product of type P_{j-4} of the k^{th} production cycle completed for j =5,6,7,8

The set of mathematical equations shown in (9) and (10) is derived for the MPPS from the concepts of max-plus algebra. For example, the event $x_1(k)$ starts at the instant of max{5+x_2(k-1), 6+x_{11}(k-1), u_6(k), u_1(k)}. It implies that the event x_1(k) starts at the *maximum* time of either of the following time instants:
the time at which the event x_2 is finished in (k-1)th cycle; the time at which the event x_{11} is finished in (k-1)th cycle; the time at which part P_2 (correspond to u_6) and machine M_1 (correspond to u_1) is available in kth cycle. It is also reflected from the timed event graph model of the system, as the events x_2 and x_{11} are input to the event x_1. Similarly procedure is adopted while deriving the equations for other events.

$$\left.\begin{aligned}
x_1(k) &= 5\otimes x_2(k-1)\oplus 6\otimes x_{11}(k-1)\oplus u_6(k)\oplus u_1(k)\\
x_2(k) &= 1\otimes x_1(k)\oplus 7\otimes x_8(k-1)\oplus u_7(k)\\
x_3(k) &= 4\otimes x_5(k-1)\oplus 4\otimes x_{10}(k-1)\oplus u_5(k)\oplus u_2(k)\\
x_4(k) &= 3\otimes x_3(k)\oplus 1\otimes x_1(k)\\
x_5(k) &= 3\otimes x_4(k)\oplus 5\otimes x_2(k)\\
x_6(k) &= 4\otimes x_9(k-1)\oplus 3\otimes x_3(k)\oplus u_3(k)\\
x_7(k) &= 5\otimes x_6(k)\oplus 3\otimes x_4(k)\\
x_8(k) &= 2\otimes x_7(k)\oplus 4\otimes x_5(k)\\
x_9(k) &= 7\otimes x_8(k)\oplus 3\otimes x_{12}(k-1)\oplus u_8(k)\\
x_{10}(k) &= 5\otimes x_6(k)\oplus 3\otimes x_{12}(k-1)\oplus u_8(k)\\
x_{11}(k) &= 2\otimes x_7(k)\oplus 4\otimes x_{10}(k)\\
x_{12}(k) &= 4\otimes x_9(k)\oplus 6\otimes x_{11}(k)
\end{aligned}\right\} \quad \text{-- (9)}$$

The equations for the finished product types can be derived as follows:

$$\left.\begin{aligned}
y_1(k) &= 5\otimes x_2(k)\\
y_2(k) &= 4\otimes x_5(k)\\
y_3(k) &= 4\otimes x_9(k)\\
y_4(k) &= 3\otimes x_{12}(k)\\
y_5(k) &= 4\otimes x_{10}(k)\\
y_6(k) &= 6\otimes x_{11}(k)\\
y_7(k) &= 7\otimes x_8(k)\\
y_8(k) &= 3\otimes x_{12}(k)
\end{aligned}\right\} \quad \text{-- (10)}$$

The evolution equations (9) and (10) of MPPS can be simplified in matrix-equation form as follows:

$$\begin{aligned}
x(k) &= A_0\otimes x(k)\oplus A_1\otimes x(k-1)\oplus B_0\otimes u(k)\\
&= A\otimes x(k-1)\oplus B\otimes u(k)
\end{aligned}$$

$$y(k) = C\otimes x(k)$$

Where:

$$A = A_0^*\otimes A_1$$
$$B = A_0^*\otimes B_0$$
$$x(k) = [x_1(k), x_2(k),...., x_{12}(k)]^T$$
$$x(k-1) = [x_1(k-1), x_2(k-1),...., x_{12}(k-1)]^T$$
$$u(k) = [u_1(k), u_2(k),...., u_8(k)]^T$$

Here A_0^* is the kleene star operator matrix of max-plus algebra [8]. MPA-algorithm is implemented for the equation set (9) and (10), and the evaluation of matrices is made using Matlab programs. It results in the final matrix Â as:

$$\hat{A} = \begin{bmatrix} \varepsilon & 5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon \\ \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 7 & \varepsilon & \varepsilon & 7 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 4 & \varepsilon & \varepsilon \\ \varepsilon & 6 & \varepsilon & \varepsilon & 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 7 & 7 & \varepsilon \\ \varepsilon & 11 & \varepsilon & \varepsilon & 10 & \varepsilon & \varepsilon & 12 & \varepsilon & 10 & 12 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 7 & \varepsilon & \varepsilon & \varepsilon & 4 & 7 & \varepsilon & \varepsilon \\ \varepsilon & 9 & \varepsilon & \varepsilon & 12 & \varepsilon & \varepsilon & \varepsilon & 9 & 12 & 10 & \varepsilon \\ \varepsilon & 15 & \varepsilon & \varepsilon & 14 & \varepsilon & \varepsilon & 16 & 11 & 14 & 16 & \varepsilon \\ \varepsilon & 22 & \varepsilon & \varepsilon & 21 & \varepsilon & \varepsilon & 23 & 18 & 21 & 23 & 3 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 12 & \varepsilon & \varepsilon & \varepsilon & 9 & 12 & \varepsilon & 3 \\ \varepsilon & 11 & \varepsilon & \varepsilon & 16 & \varepsilon & \varepsilon & \varepsilon & 13 & 16 & 12 & 7 \\ \varepsilon & 26 & \varepsilon & \varepsilon & 25 & \varepsilon & \varepsilon & 27 & 22 & 25 & 27 & 13 \end{bmatrix}$$

The calculation of unique eigen value λ (i.e. the maximum cycle mean) for the matrix Â is determined by applying the Karp's theorem which is equal to 18. It implies that the maximum cycle mean for the given MPPS is equal to 18. The throughput rate of the system is equal to the inverse of the cycle mean. Hence the throughput rate = 1/ λ = 1/18.

The periodic behavior of the MPPS has been characterized graphically by running the system for ten consecutive cycles, and a Gantt chart shown in Fig.4 is drawn with time on *x*-axis and the event number on *y*-axis. The chart indicates the state vector evolution or the occurrence of various events with time. It clearly shows the cyclic operational behavior of the resources within the system for all the events.

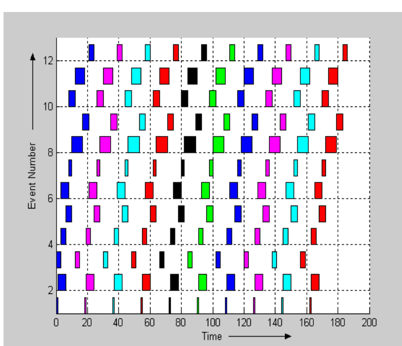

Fig 4. Gantt chart for execution of events with time

The MPPS can be treated as a flow shop problem since all the parts have the same processing sequence through the machines. Therefore other performance measures of the system are evaluated by determining the sequence of max-plus output vector *Y*. The vector *Y* shown below is determined by running the system for ten consecutive cycles ($1 \le k \le 10$). The rows indicate the values of output vector y_i ($1 \le i \le 8$) and the columns indicate the cycle number (k).

$$Y = \begin{bmatrix} 6 & 24 & 42 & 60 & 78 & 96 & 114 & 132 & 150 & 168 \\ 10 & 28 & 46 & 64 & 82 & 100 & 118 & 136 & 154 & 172 \\ 21 & 39 & 57 & 75 & 93 & 111 & 129 & 147 & 165 & 183 \\ 24 & 42 & 60 & 78 & 96 & 114 & 132 & 150 & 168 & 186 \\ 12 & 30 & 48 & 66 & 84 & 102 & 120 & 138 & 156 & 174 \\ 18 & 36 & 54 & 72 & 90 & 108 & 126 & 144 & 162 & 180 \\ 17 & 35 & 53 & 71 & 89 & 107 & 125 & 143 & 161 & 179 \\ 24 & 42 & 60 & 78 & 96 & 114 & 132 & 150 & 168 & 186 \end{bmatrix}$$

For machine i ($1 \le i \le 4$) and part j ($5 \le j \le 8$), following performance relations of flow-shop systems can be applied.

Flow time of part $j = y_j(k) - y_j(k-1)$

Average flow time for part j =

$$\frac{1}{k}\sum_{k=1}^{k} y_j(k) - y_j(k-1)$$

Average processing time of part j = $\frac{1}{k}\sum_{k=1}^{k} d_j(k)$

Average workload of machine i = $\frac{1}{k}\sum_{k=1}^{k} d_i(k)$

Average cycle time for machine i =

$$\frac{1}{k}\sum_{k=1}^{k} y_i(k) - y_i(k-1)$$

Application of these relations to the output vector *Y* values and the processing times of Table 1 results in the system performance measures shown in Table 3 and Table 4.

Table 3: Performance measures of parts

Part No. →	**1**	**2**	**3**	**4**
Flow time	18	18	18	18
Average flow time	18	18	18	18
Average processing time	12	12	16	7

Table 4: Performance measures of machines

Machine No. →	**1**	**2**	**3**	**4**
Average workload	6	10	18	13
Average cycle time	18	18	18	18

6. CONCLUSIONS

Max-plus algebra, a new mathematical modeling tool used for DEDS, have been shown to be useful in modeling and performance evaluation of production systems. The MPA-algorithm presented in this paper can be applied for other production systems. Matlab programs can be used to analyze the max-plus model of the system. It is concluded that the Matlab programs of MPA-algorithm are helpful in real time control of production systems. It is observed that the computational time required for determining the unique eigen value of the final matrix Â increases as the number of events in system increases. Further research can be carried out by exploring the potential application of MPA-algorithm for modeling of modular production systems.

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