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ANALYSIS OF MEANS FOR ANALYZING MISSING DATA FROM EXPERIMENTAL DESIGNS – PART I

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ABSTRACT

A step-by-step analysis of means (ANOM) procedure proposed by Subramani (1992) to analyze the missing data from randomized block designs has been extended to other experimental designs with several missing observations. The proposed method is general in nature. For the sake of simplicity the procedure of analysis of means to analyze missing data from experimental designs has been discussed in two parts. In part I, it is planned to apply this method for analyzing missing data from latin square designs, graeco latin square designs and hyper graeco latin square designs. The part II of this paper is dedicated to analyze the missing data from replicated latin square designs, cross over designs and F-Square designs. The procedure is also illustrated with the help of numerical examples.

Key words: Analysis of Means; Missing Data; Latin Square Designs; Graeco Latin Square Designs; Hyper Graeco-Latin-Square Designs.

1. Introduction

Analysis of Means (ANOM) introduced by ott (1967) is a graphical procedure to analyze the data from experimental designs with factors at fixed levels. Schilling (1973) has extended the ANOM procedure and introduced analysis of means for treatment effects (ANOME) to analyze the fixed effects in the crossed classifications, nested designs, balanced incomplete block designs etc., So for, the ANOME procedure has been applied only to balanced models, except the unbalanced one way fixed effects model. Subramani (1992) has applied the ANOME procedure to analyze the missing data from randomized block designs. The fundamental idea to use the ANOM procedure for analyzing the missing data is to get the complete data by inserting estimates of missing values in their respective positions. If we use least squares estimates of missing values then the resulting estimates of the treatment effects and the residual sum of squares obtained from the augmented data are respectively equal to the treatment effects and residual sum of squares obtained through the non-orthogonal data analysis of the original data. The estimates of the missing values may be obtained by using any one of the methods discussed by Subramani and Ponnusamy (1989), Wilkinson (1958) and Yates (1933).

In this part I of this paper, a step-by-step ANOME procedure is given to analyze the missing data from latin square designs, graeco latin square designs and hyper graeco latin square designs with several missing observations. The procedure is also illustrated with the help of a numerical example for each of the above cases.

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2. ANOME Procedure For Missing Data

The step-by-step ANOME procedure to analyze the missing data from any of the experimental designs is as follows:

Step 1: Write the model of the experimental design.

Step 2: Obtain the elements of the matrix A. The elements of the matrix A are obtained from the position of the missing values in the data table (Subramani and Ponnuswamy, 1989)

Step 3: Obtain the elements of the vector b.

Step 4: Obtain the estimates of the missing values using $x = A^{-1} b$ and substitute these values into the data table.

Step 5: Estimate the treatment affects using standard orthogonal methods.

Step 6 : Determine the degrees of freedom for the error sum of squares (f*, say) as f* = f-m, where f is the degrees of freedom for error sum of squares obtained from the ANOVA table with no missing values and *m* is the number of missing values.

Step 7: Obtain the estimate of the experimental error σ. Normally the estimate of σ is obtained as the square root of the error mean square from the ANOVA table.

Step 8: Determine the decision lines (LDL and UDL) for the desired α risk as

$$0 \pm \sigma h_\alpha \sqrt{q/n}$$

where n: total number of observations in the experiment.
q: degrees of freedom for the treatment effects to be plotted.
$h_\alpha : h_\alpha(k, f^*)$, critical factor obtained from the table of Schilling (1973), k is the number of points to be plotted.

Step 9: Plot the treatment effects against the decision lines and draw the statistical inference. That is, if any of the treatment effects plotted on the ANOME chart falls outside of either UDL or LDL, conclude that the treatment effects are not homogeneous. Otherwise conclude that the treatment effects are homogeneous at the given level of significance.

3. Latin Square Designs

In this section, the step by step procedure of analyzing missing data from latin square designs is presented and also illustrated with the help of a numerical example. The procedure is discussed in Section 3.1., where as the numerical example is given in Section 3.2.

3.1 ANOME to Analyze Missing Data from Latin Square Designs

Consider a latin square design with p treatments in p rows and p columns. Let m be the number of missing values then $m < (p-1)(p-2)$. The proposed ANOME procedure is given below:
Step 1: The model of a latin square design is

$$Y_{ijk} = \mu + r_i + c_j + t_k + e_{ijk}, \qquad i, j, k = 1,2,..., p$$

where $Y_{ij(k)}$ is the observation from i^th^ row, j^th^ column and k^th^ treatment; r_i is the effect of i^th^ row; c_j is the effect of j^th^ column; t_k is the effect of k^th^ treatment; and $e_{ij(k)}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani and Ponnuswamy (1989) as

$$A = (a_{ij}) = \begin{cases} (p-1)(p-2) & \text{if } i = j \\ -(p-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular row or column or treatment} \\ 2 & \text{Otherwise} \end{cases}$$

Step 3: The elements of the vector b are obtained as

$$b_i = p(R_{(i)} + C_{(i)} + T_{(i)}) - 2G'$$

where $R_{(i)}$, $C_{(i)}$ and $T_{(i)}$ are respectively the row, column and treatment totals corresponding to the i^th^ missing value and G' is the grand total of all known observations.

Step 4: The estimates of the missing values are obtained as $\boldsymbol{x = A^{-1} b}$

Step 5: Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects. The k^th^ treatment effect is obtained as

$$T_k = \bar{Y}_{..(k)} - \bar{Y}, \text{ k=1,2, …, p.}$$

where $\bar{Y}_{..(k)}$ is the mean value of the k^th^ treatment and $\bar{Y}$ is the grand mean.

Step 6: Determine the degrees of freedom for error sum of squares as f* =f-m, where f = (p-1) (p-2).

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ijk}^2 - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{k=1}^{p} T_k^2 + \frac{2}{p^2} G^2 \right) / f^*}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk ∂ as follows:

$0 \pm \sigma h_\alpha \sqrt{(p-1)/p^2}$, where $h_\alpha = h_\alpha(p, f^*)$

Step 9: Plot the treatment effects $T_1, T_2, \ldots, T_p$ against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous.

3.2 Numerical Example

Consider the Latin square design given in Subramani and Ponnuswamy (1989) with 5 treatments. In the data given in Table 3.1 the observations Y_{13}, Y_{34} and Y_{42} are the missing values. The resulting data are given below:

Table 3.1: Missing data in LSD

50(A)	64(B)	**(C)	65(D)	59(E)
70(B)	76(C)	65(D)	62(E)	60(A)
60(C)	59(D)	59(E)	**(A)	62(B)
52(D)	**(E)	60(A)	60(B)	60(C)
64(E)	68(A)	58(B)	62(C)	63(D)

Let X_1, X_2 and X_3 be the corresponding estimated values of the missing values Y_{13}, Y_{34} and Y_{42}. By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 12 & 2 & 2 \\ 2 & 12 & 2 \\ 2 & 2 & 12 \end{bmatrix}$$

b_1 = 5(238 + 242 + 258) – 2 x 1358 = 974
b_2 = 5(240 + 249 + 238) – 2 x 1358 = 919
b_3 = 5(232 + 267 + 244) – 2 x 1358 = 999

Since the missing values are of a particular pattern discussed in Subramani and Ponnuswamy (1989), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{[(p-1)(p-2)+2(m-1)]b_i - 2\sum_{j=1}^{m} b_j}{[(p-1)(p-2)-2][(p-1)(p-2)+2(m-1)]}$$

, i = 1, 2, ….., m

It is given that m=3 and p=5. By substituting those values in the above equation the estimates of the missing values are obtained as:

$X_1 = Y_{13} = 61.25$
$X_2 = Y_{34} = 55.75$ and
$X_3 = Y_{42} = 63.75$

After substituting the estimated values of the missing values in their respective positions the treatment effects are obtained as:

T1 = 58.75 – 61.55 = -2.80
T2 = 62.80 – 61.55 = 1.25
T3 = 63.85 – 61.55 = 2.30
T4 = 60.80 – 61.55 = -0.75 and
T5 = 61.55 – 61.55 = 0.00

The degrees of freedom for error component σ^2 is f* = 12-3 = 9

The estimate of the experimental error $\hat{\sigma}$ is obtained as $\hat{\sigma}$ = 5.155

The value of the critical factor is obtained as $h_{o.05}$ (5, 9) = 3.25

The decision lines at α= 0.05 are obtained as

LDL = 0-3.25*5.55 * $\sqrt{4/25}$ = - 7.215

UDL = 0+3.25*5.55 * $\sqrt{4/25}$ = 7.215

Now plot the treatment effects T_1, T_2, T_3, T_4 and T_5 as in Figure 3.1. From the Figure 3.1 it is observed that all the plotted treatment effects are within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.

4. Graeco Latin Square Designs

In this section, the step by step procedure of analyzing missing data from graeco latin square designs is presented and also illustrated with the help of a numerical example. The procedure is discussed in Section 4.1., where as the numerical example is given in Section 4.2.

4.1 ANOME to Analyze Missing Data from Graeco Latin Square Designs

Consider a graeco latin square design with p treatments of type I, p treatments of type II in p rows and p columns. For want of space, we are not discussing the definition, design and applications of graeco latin square designs and the readers are referred to Montgomery (1984) and Subramani (1991) and the references cited therein. Let m be the number of missing values then $m < (p-1)(p-3)$.

The proposed ANOME procedure is given below:

Step 1: The model of a graeco latin square design is

$$Y_{ij(kl)} = \mu + r_i + c_j + t_k + g_l + e_{ij(kl)}, \qquad i, j, k, l = 1, 2, ..., p$$

where $Y_{ij(kl)}$ is the observation from i^{th} row, j^{th} column and k^{th} treatment of type I and l^{th} treatment of type II; r_i is the effect of i^{th} row; c_j is the effect of j^{th} column; t_k is the effect of k^{th} treatment of type I; g_l is the effect of l^{th} treatment of type II and $e_{ij(kl)}$ is the error component with mean 0 and variance σ^2.

Step 2: The elements of the matrix A are obtained from Subramani (1991) as

$$A = (a_{ij}) = \begin{cases} (p-1)(p-3) & \text{if } i = j \\ -(p-3) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular row or column or treatment of type I or type II} \\ 3 & \text{Otherwise} \end{cases}$$

Step 3: The elements of the vector b are obtained as

$$b_i = p(R_{(i)} + C_{(i)} + T_{(i)} + G_{(i)}) - 3G'$$

where $R_{(i)}$, $C_{(i)}$, $T_{(i)}$ and $G_{(i)}$ are respectively the row, column and treatment totals of type I and type II corresponding to the i^{th} missing value and G' is the grand total of all known observations.

Step 4: The estimates of the missing values are obtained as $\boldsymbol{x = A^{-1} b}$

Step 5: Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects of type I and type II. The k^{th} treatment effect of type I and l^{th} treatment effect of type II respectively are obtained as

$$T_k = \overline{Y}_{..(k.)} - \overline{Y}, \text{ k=1, 2, …, p.}$$

$$G_l = \overline{Y}_{..(.l)} - \overline{Y}, \text{ l=1, 2, … , p}$$

where $\overline{Y}_{..(k.)}$, is the mean value of the k^{th} treatment of type I, $\overline{Y}_{..(.l)}$ is the mean value of the l^{th} treatment of type II and $\overline{Y}$ is the grand mean.

Step 6: Determine the degrees of freedom for error sum of squares as f* =f-m, where f = (p-1) (p-3).

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y^2_{ij(kl)} - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{k=1}^{p} T_k^2 - \frac{1}{p}\sum_{l=1}^{p} G_l^2 + \frac{3}{p^2}G^2\right) / f^*}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma h_\alpha \sqrt{(p-1)/p^2}, \text{ where } h_\alpha = h_\alpha(p, f^*)$$

Step 9: Plot the treatment effects T_1, T_2, …., T_p and G_1, G_2, …., G_p against their respective decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous at α level of significance.

4.2 Numerical Example

Consider the graeco latin square design with 5 treatments (Type I and Type II) in 5 rows (batches) and 5 columns (acid concentrations) given in Montgomery (1984, p.163). In the data, assume that the observations Y_{12}, Y_{13} and Y_{14} are missing. The resulting data are given below:

Table 4.1: Missing Data in GLSD

Batch	Acid Concentration					Total
	1	2	3	4	5	
1	26 (Aα)	**(Bβ)	**(Cγ)	**(Dδ)	13 (Eε)	39
2	18 (Bγ)	21(Cδ)	18(Dε)	11(Eα)	21 (Aβ)	89
3	20 (Cε)	12 (Dα)	16 (Eβ)	25 (Aγ)	13 (Bδ)	86
4	15 (Dβ)	15 (Eγ)	22 (Aδ)	14 (Bε)	17 (Cα)	83
5	10 (Eδ)	24 (Aε)	17 (Bα)	17 (Cβ)	14 (Dγ)	82
Total	89	72	73	67	78	379

Let X_1, X_2 and X_3 be the corresponding estimated values of the missing values Y_{12}, Y_{13} and Y_{14}. By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix}$$

b_1 = 5(39 + 72 + 62 + 69) – 3 x 379 = 73
b_2 = 5(39 + 73 + 75 + 72) – 3 x 379 = 158
b_3 = 5(39 + 67 + 59 + 66) – 3 x 379 = 18

Since the missing values are of a particular pattern discussed in Subramani (1991), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{(p-m)b_i + \sum_{j=1}^{m} b_j}{p(p-3)(p-m)}, \quad i = 1, 2, \ldots, m.$$

It is given that m=3 and p=5. By substituting those values in the above equation, the estimates of the missing values are obtained as:

$$X_1 = Y_{12} = \frac{(5-3)*73 + (73+158+18)}{5(5-3)(5-3)} = \frac{395}{20} = 19.75$$

$$X_2 = Y_{13} = \frac{(5-3)*158 + (73+158+18)}{5(5-3)(5-3)} = \frac{565}{20} = 28.25$$

$$X_3 = Y_{14} = \frac{(5-3)*18 + (73+158+18)}{5(5-3)(5-3)} = \frac{285}{20} = 14.25$$

After substituting the estimated values of the missing values in there respective positions the treatment effects of type I are obtained as:

T1 = 23.60 – 17.65 = 5.95
T2 = 16.35 – 17.65 = -1.30
T3 = 20.65 – 17.65 = 3.00
T4 = 14.65 – 17.65 = -3.00 and
T5 = 13.00 – 17.65 = -4.65

Similarly the treatment effects of type II are obtained as:

G1 = 16.60 – 17.65 = -1.05
G2 = 17.75 – 17.65 = 0.10
G3 = 20.05 – 17.65 = 2.40
G4 = 16.05 – 17.65 = -1.60 and
G5 = 17.80 – 17.65 = 0.15

The degrees of freedom for error component σ^2 is f* = 8-3 = 5

The estimate of the experimental error $\hat{\sigma}$ is obtained as $\hat{\sigma}$ = 1.7804

The value of the critical factor is obtained as $h_{o.05}$ (5,5) = 4.04

The decision lines at α= 0.05 are obtained as

LDL = 0-1.7804*4.04* $\sqrt{4/25}$ = - 2.877126

UDL = 0+1.7804*4.04* $\sqrt{4/25}$ = 2.877126

Now plot the treatment effects T_1, T_2, T_3, T_4 and T_5 and G_1, G_2, G_3, G_4 and G_5 as in Figure 4.1 and Figure 4.2 respectively. From the Figure 4.1 it is observed that all the plotted treatment effects, except T_2, fall outside the decision lines. Hence one can conclude that the effects of different treatments of Type –I are not the same at 5% level of significance. Similarly, from the Figure 4.1 it is observed that all the plotted treatment effects fall inside the decision lines. Hence one can conclude that the effects of different treatments of Type –II are the same at 5% level of significance

5. Hyper Graeco Latin Square Designs

In this section, the step by step procedure of analyzing missing data from hyper graeco latin square designs is presented and also illustrated with the help of a numerical example. The procedure is discussed in Section 5.1., where as the numerical example is given in Section 5.2.

5.1 ANOME to Analyze Missing Data from Hyper Graeco Latin Square Designs

Consider a k^{th} order hyper graeco latin square design with p treatments in each of k-types in p rows and p columns. For want of space, we are not discussing the definition, design and applications of hyper graeco latin square designs and the readers are referred to Chakrabarti (1962) and Subramani (1993a). Let m be the number of missing values then $m < (p-1)(p-(k+1))$. The proposed ANOME procedure is given below:

Step 1: The model of a graeco latin square design is

$$Y_{ij(l(s))} = \mu + r_i + c_j + \sum_{s=1}^{k} t_{l(s)} + e_{ij(l(s))}, \qquad i, j, l = 1,2,\ldots, p$$

where $Y_{ij(l(s))}$ is the observation from i^{th} row, j^{th} column and l^{th} treatment of type S; r_i is the effect of i^{th} row; c_j is the effect of j^{th} column; $t_{l(s)}$ is the effect of l^{th} treatment of type S and $e_{ij(l(s))}$ is the error component with mean 0 and variance σ^2.

Step 2: The elements of the matrix A are obtained from Subramani (1993a) as

$$A = (a_{ij}) = \begin{cases} (p-1)(p-(k+1)) & \text{if } i = j \\ -(p-(k+1)) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular row or column or treatment of type S} \\ (k+1) & \text{Otherwise} \end{cases}$$

Step 3: The elements of the vector b are obtained as

$$b_i = p(R_{(i)} + C_{(i)} + \sum_{s=1}^{k} T_{s(i)}) - (k+1)G'$$

where $R_{(i)}$, $C_{(i)}$ and $T_{s(i)}$ are respectively the row, column and treatment totals of type S corresponding to the i[th] missing value and G' is the grand total of all known observations.

Step 4: The estimates of the missing values are obtained as ***x = A^-1^ b***

Step 5: Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects of type S. The i[th] treatment effect of type S is obtained as

$T_{i(s)} = \overline{Y}_{..(i(s))} - \overline{Y}$, i=1, 2, …, p and s=1, 2, …, k

Where $\overline{Y}_{..(i(s))}$ is the mean value of the i[th] treatment of type S and $\overline{Y}$ is the grand mean

Step 6: Determine the degrees of freedom for error sum of squares as f* =f-m, where

f = (p-1) (p-(k+1)).

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(kl)}^2 - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{s=1}^{k}\sum_{l}^{p} T_{l(s)}^2 + \frac{(k+1)}{p^2} G^2 \right) / f^*}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma\, h_\alpha \sqrt{(p-1)/p^2}\ ,$$ where

$$h_\alpha = h_\alpha(p, f^*)$$

Step 9: Plot the treatment effects $T_{1(s)}$, $T_{2(s)}$, …., $T_{p(s)}$ of treatment Type S for s=1, 2, …, k against their respective decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous at α level of significance.

5.2 Numerical Example

Consider the data given in Table 5.1 as of a hyper graeco latin square design of 3[rd] order with 7 treatments of Type-I (A, B, C, D, E, F, G, H), Type-II (1, 2, 3, 4, 5, 6, 7) and Type-III (a, b, c, d, e, f, g) in 7 rows and 7 columns. In the data, assume that the observations Y_{11}, Y_{22} and Y_{33} are missing. The resulting data are given below:

Table 5.1: Missing Data in HGLSD

Row	Column						
	1	2	3	4	5	6	7
1	**(A1a)	66(E3d)	56(B5g)	52(F7c)	61(C2f)	65(G4b)	58(D6e)
2	64(E7f)	**(B2b)	50(F4e)	64(C6a)	63(G1d)	64(D3g)	63(A5c)
3	69(B6d)	53(F1g)	**(C3c)	61(G5f)	67(D7b)	64(A2e)	59(E4a)
4	57(F5b)	58(C7e)	67(G2a)	65(D4d)	55(A6g)	58(E1c)	60(B3f)
5	67(C4g)	57(G6c)	66(D1f)	60(A3b)	57(E5e)	62(B7a)	64(F2d)
6	62(G3e)	59(D5a)	62(A7d)	63(E2g)	60(B4c)	66(F6f)	62(C1b)
7	54(D2c)	60(A4f)	58(E6b)	65(B1e)	59(F3a)	70(C5d)	64(G7g)

That is, p=7, k=3 and m=3. From the above table, one can obtain the row, column and treatment totals as given below:

Category	1	2	3	4	5	6	7	Total
Row	358	368	373	420	433	434	430	2816
Column	373	353	359	430	422	449	430	2816
Treat.-I	364	372	382	433	425	401	439	2816
Treat.-II	367	373	371	426	423	427	429	2816
Trat.-III	370	369	344	459	414	438	422	2816
Total	1832	1835	1829	2168	2117	2149	2150	

Let X_1, X_2 and X_3 be the corresponding estimated values of the missing values Y_{11}, Y_{22} and Y_{33}. By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 18 & 4 & 4 \\ 4 & 18 & 4 \\ 4 & 4 & 18 \end{bmatrix}$$

b_1 = 7(358 + 373 + 364 + 367 + 370) – 4 x 2816 = 1560
b_2 = 7(368 + 353 + 372 + 373 + 369) – 4 x 2816 = 1581
b_3 = 7(373 + 359 + 382 + 371 + 344) – 4 x 2816 = 1539

Since the missing values are of a particular pattern discussed in Subramani (1993a), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{\left[(p-1)[p-(k+1)]+(m-1)(k+1)\right]b_i - (k+1)\sum_{j=1}^{m} b_j}{\left[(p-1)[p-(k+1)]+(m-1)(k+1)\right]\left[(p-1)[p-(k+1)]-(k+1)\right]}, \qquad i = 1, 2, \ldots, m.$$

It is given that k=3, m=3 and p=7, $b_1 = 1560, b_2 = 1581 \, and \, b_3 = 1539$. By substituting these values in the above equation, the estimates of the missing values are obtained as:

$$X_1 = Y_{11} = \frac{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]1560-(3+1)[1560+1581+1539]}{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]\left[(7-1)[7-(3+1)]-(k+1)\right]} = \frac{40560-18720}{364} = \frac{21840}{364} = 60$$

$$X_2 = Y_{22} = \frac{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]1581-(3+1)[1560+1581+1539]}{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]\left[(7-1)[7-(3+1)]-(k+1)\right]} = \frac{41106-18720}{364} = \frac{22386}{364} = 61.5$$

$$X_3 = Y_{33} = \frac{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]1539-(3+1)[1560+1581+1539]}{\left[(7-1)[7-(3+1)]+(3-1)(3+1)\right]\left[(7-1)[7-(3+1)]-(k+1)\right]} = \frac{40014-18720}{364} = \frac{21294}{364} = 58.5$$

After substituting the estimated values of the missing values in there respective positions the treatment effects of Type-I are obtained as:

T_A = 424/7-2996/49 = -0.57
T_B = 424/7-2996/49 = 0.786
T_C = 440.5/7-2996/49 =1.786
T_D = 433/7-2996/49 = 0.714
T_E = 425/7-2996/49 = 0.43

T_F = 401/7-2996/49 = -3.86 and
T_G = 439/7-2996/49 = 1.571

Similarly the treatment effects of Type-II and of Type-III are obtained as:

T_1 = 427/7-2996/49 = -0.14
T_2 = 434.5/7-2996/49 = 0.929
T_3 = 429.5/7-2996/49 = 0.214
T_4 = 426/7-2996/49 = -0.29
T_5 = 423/7-2996/49 = -0.71
T_6 = 427/7-2996/49 = -0.14 and
T_7 = 429/7-2996/49 = 0.143

T_a = 430/7-2996/49 = 0.286
T_b = 430.5/7-2996/49 = 0.357
T_c = 402.5/7-2996/49 =-3.64
T_d = 459/7-2996/49 = 4.429
T_e = 414/7-2996/49 = -2.000
T_f = 438/7-2996/49 = 1.429 and
T_g = 422/7-2996/49 = 0.86

The degrees of freedom for error component σ^2 is f* = 18-3 = 15
The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(kl)}^2 - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{s=1}^{k}\sum_{l}^{p} T_{l(s)}^2 + \frac{(k+1)}{p^2} G^2\right) / f^*}$$

$$\hat{\sigma} = \sqrt{\left(184112.5 - \frac{1282531.5}{7} - \frac{1283090.5}{7} - \frac{1282364.5}{7} - \frac{128364.5}{7} - \frac{1284241.5}{7} + \frac{(3+1)8976016}{49}\right)/15}$$

$$\hat{\sigma} = 4.7177$$

The value of the critical factor is obtained as $h_{o.05}$ (7, 15) = 3.11

The decision lines at α= 0.05 are obtained as

LDL = 0- 4.7177*3.11* $\sqrt{6/49}$ = -5.13414

UDL = 0+4.7177*3.11* $\sqrt{6/49}$ = 5.13414

Now plot the treatment effects of Type-I, T_A, T_B, T_C, T_D, T_E, T_F and T_G, of Type-II, T_1, T_2, T_3, T_4, T_5, T_6 and T_7, and of Type-III, T_a, T_b, T_c, T_d, T_e, T_f and T_g, as in Figure 5.1, Figure 5.2 and Figure 5.3 respectively. From the Figures 5.1-5.3, it is observed that all the plotted treatment effects of Type-I, Type-II and Type-III are within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.

6.Summary

It is well known that the ANOME procedure is useful to assess the engineering significance as well as statistical significance from the experimental data with factors at fixed levels. However, the drawback of this procedure is that, it is used so far only for the balanced experimental designs. That is, the ANOME Procedure is applicable only if we have equal number of observations in each cell of the experimental designs. In this paper a step-by-step method is presented for the use of ANOME procedure to analyze the Latin square designs, Graeco Latin Square Designs and Hyper Graeco Latin Square Designs with several missing observations. The procedure is also illustrated with the help of numerical examples. The key point in using the ANOME procedure to analyze the missing data is to get complete data by inserting estimates of the missing values in their respective cells. For estimating several missing values from graeco-latin square designs, hyper-graeco- latin square designs, crossover designs and F-square designs one may refer to Subramani (1991,93,94) and Subramani and Aggarwal (1993).

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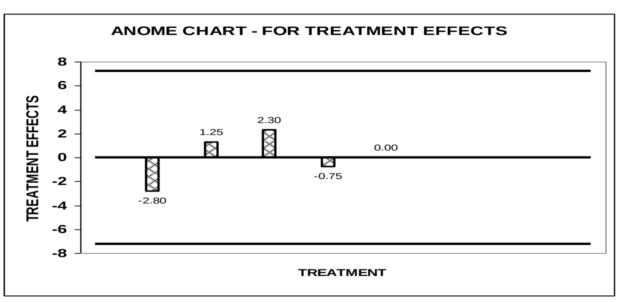

Fig.3.1 Anome Chart For Treatment

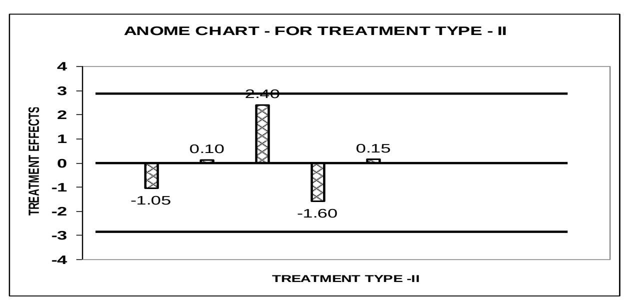

Fig.4.2 Anome Chart For Treatment Type – II

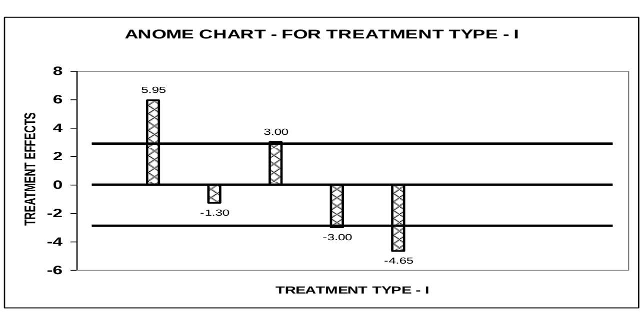

Fig. 4.1 Anome Chart For Treatment Type – I

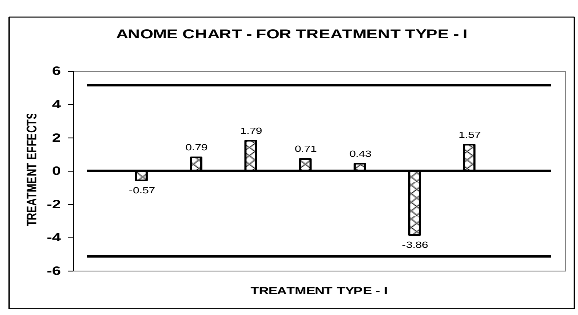

Fig. 5.1 Anome Chart For Treatment Type I

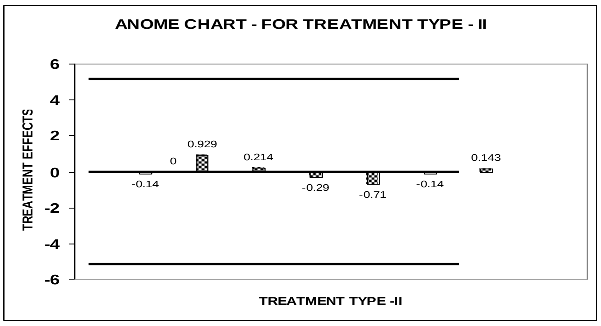

Fig. 5.2 Anome Chart For Treatment Type –II

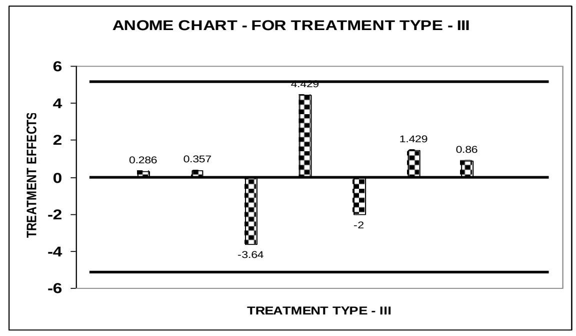

Fig. 5.3 Anome Chart For Treatment Type - III