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ANALYSIS OF MEANS FOR ANALYZING MISSING DATA FROM EXPERIMENTAL DESIGNS – PART II

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ABSTRACT

A step-by-step analysis of means (ANOM) procedure proposed by Subramani (1992) to analyze the missing data from randomized block designs has been extended to other experimental designs with missing observations. In part I of this paper, we have applied this method for analyzing the missing data from latin square designs, graeco-latin square designs and hyper graeco -latin square designs. In part II of this paper, it is decided to analyze the missing data from replicated latin square designs of Type I , Type II and Type III, cross over designs and F-Square designs. The procedure is also illustrated with the help of numerical examples.
Key words: Analysis of Means; Missing Data; Replicated Latin Square Designs; Cross Over Designs; F-Square Designs.

1. Introduction

Analysis of Means (ANOM) introduced by Ott (1967) is a graphical procedure to analyze the data from experimental designs with factors at fixed levels. Schilling (1973) has extended the ANOM procedure and introduced analysis of means for treatment effects (ANOME) to analyze the fixed effects in the crossed classifications, nested designs, balanced incomplete block designs etc. Subramani (1992) has applied the ANOME procedure to analyze the missing data from randomized block designs. This procedure has been extended by Subramani (2008a) to Latin Square Designs, Graeco Latin Square Designs and Hyper Graeco Latin Squares Designs. The fundamental idea is to get the estimates of the missing values, which may be obtained by using any one of the methods discussed by Subramani and Ponnuswamy (1989), Wilkinson (1958) and Yates (1933).

In this part II of this paper, a step-by-step ANOME procedure is given to analyze the missing data from Replicated Latin Squares Designs of Type I, Type II and Type III, Cross-over Designs without Residual Effects and F-Square Designs. The procedure is also illustrated with the help of a numerical example for each of the above designs.

2. ANOME Procedure For Missing Data

For the sake of convenience and easy reference to the readers, the step-by-step ANOME procedure to analyze the missing data from any of the experimental designs is given below:

Step 1 : Write the model of the experimental design

Step 2 : Obtain the elements of the matrix A. The elements of the matrix A are obtained from the position of the missing values in the data table (Subramani and Ponnuswamy, 1989)

Step 3 : Obtain the elements of the vector b

Step 4 : Obtain the estimates of the missing values using $x = A^{-1} b$ and substitute these values into the data table

Step 5 : Estimate the treatment affects using standard orthogonal methods

Step 6 : Determine the degrees of freedom for the error sum of squares (f*, say) as f* = f-m, where f is the degrees of freedom for error sum of squares obtained from the ANOVA table with no missing values and *m* is the number of missing values

Step 7 : Obtain the estimate of the experimental error σ. Normally the estimate of σ is obtained as the square root of the error mean square from the ANOVA table.

Step 8 : Determine the decision lines (LDL and UDL) for the desired α risk as

$$0 \pm \sigma h_{\alpha} \sqrt{q/n}$$

Where n: total number of observations in the experiment. q: degrees of freedom for the treatment effects to be plotted. $h_{\alpha} : h_{\alpha}(k, f^*)$, critical factor obtained from the table of Schilling (1973), k is the number of points to be plotted.

Step 9 : Plot the treatment effects against the decision lines and draw the statistical inference. That is, if any of the treatment effects plotted on the ANOME chart falls outsides of either UDL or LDL, conclude that the

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treatment effects are not homogeneous. Otherwise conclude that the treatment effects are homogeneous at the given level of significance.

3. Replicated Latin Squares Designs

As stated by Montgomery (1984), the disadvantage of latin squares of order 2, 3 and 4 is that they provide a small error degrees of freedom. In the case of missing values the situation becomes much worse. In such situations, one can replicate the latin square design to increase the error degrees of freedom. The replication of a latin square design will lead to the following three designs:

1. Use the same rows and the same columns in each replicate
2. Use the same rows (columns) but different columns (rows) in each replicate
3. Use different rows and different columns in each replicate

The resulting designs for the above three cases are called Replicated Latin Squares Designs of Type I, Type II and Type III respectively. For other details and for the analysis of variance table one may refer to Montgomery (1984) and Subramani (1991). In this section, the step by step ANOME procedure of analyzing missing data from Replicated Latin Squares Designs of Type I, Type II and Type III are presented and also illustrated with the help of numerical examples for each of the designs. The ANOME procedure for analyzing missing data from replicated latin squares designs are presented in Sections 3.1 to 3.3., where as the numerical examples are given in Section 3.4.

3.1. ANOME to Analyze Missing Data from Replicated Latin Squares Designs of Type I

The replicated latin squares designs of Type I of order p are obtained by replicating a latin square design of order p in n times and using the same rows and same columns in each replicate. Let m be the number of missing values then $m < (p-1)(np+n-3)$. The proposed ANOME procedure is given below:

Step 1: The model of a replicated latin squares design of Type I is

$$Y_{ij(k)l} = \mu + r_i + c_j + t_k + s_l + e_{ij(k)l}, \qquad i, j, k = 1,2,..., p,\ l = 1,2, ..., n$$

where $Y_{ij(k)l}$ is the observation from i^{th} row, j^{th} column and k^{th} treatment of l^{th} square; r_i is the effect of i^{th} row; c_j is the effect of j^{th} column; t_k is the effect of k^{th} treatment; s_l is the effect of l^{th} square and $e_{ij(k)l}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani (1991) as

$$A = (a_{ij}) = \begin{cases} (p-1)(np+n-3) & \text{if } i = j \\ -(p+n-3) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular row or column or treatment of a particular latin square} \\ -(p-3) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular row or column or treatment but from different latin squares} \\ -(n-3) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular latin square but from different rows, columns and treatments} \\ 3 & \text{Otherwise} \end{cases}$$

Step 3 : The elements of the vector b are obtained as

$$b_i = p(R_{(i)} + C_{(i)} + T_{(i)}) + nS_{(i)} - 3G',$$

i= 1, 2, 3, … ,m.

where $R_{(i)}$, $C_{(i)}$, $T_{(i)}$ and $S_{(i)}$ are respectively the row, column, treatment and latin square totals corresponding to the i^{th} missing value and G' is the grand total of all known observations.

Step 4 : The estimates of the missing values are obtained as $\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

Step 5 : Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects. The k^{th} treatment effect is obtained as

$$T_k = \bar{Y}_{(k)} - \bar{Y}, \text{ k=1,2, …, p.}$$

where $\bar{Y}_{..(k)}$ is the mean value of the k^{th} treatment and $\bar{Y}$ is the grand mean

Step 6: Determine the degrees of freedom for error sum of squares as $f^* = f - m$,

where $f = (p-1)(np+n-3)$.

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{l=1}^{n} y_{ij(k)l}^2 - \frac{1}{np}\sum_{i=1}^{p} R_i^2 - \frac{1}{np}\sum_{j=1}^{p} C_j^2 - \frac{1}{np}\sum_{k=1}^{p} T_k^2 - \frac{1}{p^2}\sum_{l=1}^{n} S_l^2 + \frac{3}{np^2}G^2\right) / f^*\right\}}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma\, h_\alpha \sqrt{(p-1)/np^2},$$

where $h_\alpha = h_\alpha(p, f^*)$

Step 9: Plot the treatment effects $T_1, T_2, …., T_p$ against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous.

3.2. ANOME to Analyze Missing Data from Replicated Latin Squares Designs of Type II

The replicated latin squares designs of Type II of order p are obtained by replicating a latin square design of order p in n times and using the same columns (rows) but different rows (columns) in each replicate. Let m be the number of missing values then $m < (p-1)(np-2)$. The proposed ANOME procedure is given below:

Step 1: The model of a replicated latin squares design of Type II is

$$Y_{ij(k)l} = \mu + r_{i(l)} + c_j + t_k + s_l + e_{ij(k)l}, \qquad i, j, k = 1,2,...,p,\ l = 1,2,\ ...,n$$

where $Y_{ij(k)l}$ is the observation from i^{th} row, j^{th} column and k^{th} treatment of l^{th} square; $r_{i(l)}$ is the effect of i^{th} row of l^{th} latin square; c_j is the effect of j^{th} column; t_k is the effect of k^{th} treatment; s_l is the effect of l^{th} latin square and $e_{ij(k)l}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani (1991) as

$$A = (a_{ij}) = \begin{cases} (p-1)(np-2) & if\ i = j \\ -(np-2) & if\ i^{th}\ and\ j^{th}\ missing\ values\ are\ of\ a\ particular\ row \\ -(p-2) & if\ i^{th}\ and\ j^{th}\ missing\ values\ are\ of\ a\ particular\ column\ or\ treatment \\ 2 & otherwise \end{cases}$$

Step 3 : The elements of the vector b are obtained as

$$b_i = p(nR_{(i)} + C_{(i)} + T_{(i)}) - 2G', \ i = 1, 2, 3, \ldots, m.$$

where $R_{(i)}$, $C_{(i)}$ and $T_{(i)}$ are respectively the row, column and treatment totals corresponding to the i^{th} missing value and G' is the grand total of all known observations.

Step 4 : The estimates of the missing values are obtained as $\boldsymbol{x = A^{-1} b}$

Step 5 : Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects. The k^{th} treatment effect is obtained as

$$T_k = \bar{Y}_{(k)} - \bar{Y}, \ k=1,2, \ldots, p.$$

where $\bar{Y}_{(k)}$ is the mean value of the k^{th} treatment and $\bar{Y}$ is the grand mean

Step 6: Determine the degrees of freedom for error sum of squares as $f^* = f - m$,

where $f = (p-1)(np-2)$.

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{ \left(\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{l=1}^{n} y_{ij(k)l}^2 - \frac{1}{p}\sum_{i=1}^{p}\sum_{l=1}^{n} R_{i(l)}^2 - \frac{1}{np}\sum_{j=1}^{p} C_j^2 - \frac{1}{np}\sum_{k=1}^{p} T_k^2 + \frac{2}{np^2} G^2 \right) / f^* \right\}}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma h_\alpha \sqrt{(p-1)/np^2}, \quad \text{where} \quad h_\alpha = h_\alpha(p, f^*)$$

Step 9: Plot the treatment effects T_1, T_2, …., T_p against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous.

3.3. ANOME to Analyze Missing Data from Replicated Latin Squares Designs of Type III

The replicated latin squares designs of Type III of order p are obtained by replicating a latin square design of order p in n times and using different rows and different columns in each replicate. Let m be the number of missing values then $m < (p-1)(np-n-1)$. The proposed ANOME procedure is given below:

Step 1: The model of a replicated latin squares design of Type III is

$$Y_{ij(k)l} = \mu + r_{i(l)} + c_{j(l)} + t_k + s_l + e_{ij(k)l}, \qquad i, j, k = 1,2,...,p,\ l = 1,2,\ ...,n$$

where $Y_{ij(k)l}$ is the observation from i^{th} row, j^{th} column and k^{th} treatment of l^{th} square; $r_{i(l)}$ is the effect of i^{th} row of l^{th} latin square; $c_{j(l)}$ is the effect of j^{th} column of l^{th} latin square; t_k is the effect of k^{th} treatment; s_l is the effect of l^{th} latin square and $e_{ij(k)l}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani (1991) as

$$A=(a_{ij})=\begin{cases}(p-1)(np-n-1) & \text{if } i=j\\ -(np-n-1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are in a particular row or column}\\ -(p-n-1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are in a particular treatment and of a particular latin square}\\ -(p-1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are in a particular treatment but from different latin squares}\\ (n+1) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular latin square but from different rows, columns and treatments}\\ 1 & \text{otherwise}\end{cases}$$

Step 3 : The elements of the vector b are obtained as $b_i = p(nR_{(i)} + nC_{(i)} + T_{(i)}) - nS_{(i)} - 2G'$, i= 1, 2, 3, … ,m. where $R_{(i)}$, $C_{(i)}$, $T_{(i)}$ and $S_{(i)}$ are respectively the row, column, treatment and latin square totals corresponding to the ith missing value and G' is the grand total of all known observations.

Step 4 : The estimates of the missing values are obtained as ***x = A^{-1} b***

Step 5 : Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects. The kth treatment effect is obtained as $T_k = \bar{Y}_{(k)} - \bar{Y}$, k=1,2, …, p.where $\bar{Y}_{(k)}$ is the mean value of the kth treatment and $\bar{Y}$ is the grand mean

Step 6: Determine the degrees of freedom for error sum of squares as $f^* = f - m$,

where $f = (p-1)(np-n-1)$.

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{l=1}^{n} y_{ij(k)l}^2 - \frac{1}{p}\sum_{i=1}^{p}\sum_{l=1}^{n} R_{i(l)}^2 - \frac{1}{p}\sum_{j=1}^{p}\sum_{l=1}^{n} C_{j(l)}^2 - \frac{1}{np}\sum_{k=1}^{p} T_k^2 + \frac{1}{p^2}\sum_{l=1}^{n} S_l^2 + \frac{1}{np^2}G^2\right)/f^*\right\}}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma h_\alpha \sqrt{(p-1)/np^2},$$

where $h_\alpha = h_\alpha(p, f^*)$

Step 9: Plot the treatment effects $T_1, T_2, \ldots, T_p$ against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous.

3.4. Numerical Example

Consider the hypothetical data given below for replicated latin squares designs with 4 treatments in 3 latin squares. That is p=4 and n=3. In the data given in Table 3.1, we get the following:

- Replicated Latin squares Design of Type I if we assume that the rows and the columns are the same for all latin squares
- Replicated Latin squares Design of Type II if we assume that the rows are different but the columns are the same for all latin squares
- Replicated Latin squares Design of Type III if we assume that the rows and the columns are different for all latin squares

For the sake of convenience the treatments A, B, C, D etc. are referred as 1, 2, 3, 4 etc. Here the observations $Y_{12(4)3}$, $Y_{23(4)3}$ and $Y_{34(4)3}$ are the missing values. The resulting data are given below:

Table 3.1.: Missing data in Replicated LSD

Latin Square 1

	C1	C2	C3	C4	Total
R1	C	D	A	B	
	10	14	7	8	39
R2	B	C	D	A	
	7	18	11	8	44
R3	A	B	C	D	
	5	10	11	9	35
R4	D	A	B	C	
	10	10	12	14	46
Total	32	52	41	39	164

Latin Square 2

	C1	C2	C3	C4	Total
R1	C	D	A	B	
	8	10	9	8	35
R2	B	C	D	A	
	11	10	8	11	40
R3	A	B	C	D	
	10	8	8	12	38
R4	D	A	B	C	
	12	10	9	8	39
Total	41	38	34	39	152

Latin Square 3

	C1	C2	C3	C4	Total
R1	C	D	A	B	
	11	***	14	10	35
R2	B	C	D	A	
	8	12	***	12	32
R3	A	B	C	D	
	9	11	12	***	32
R4	D	A	B	C	
	12	8	12	9	41
Total	40	31	38	31	140

Let X_1, X_2 and X_3 be the corresponding estimated values of the missing values $Y_{12(4)3}$, $Y_{23(4)3}$ and $Y_{34(4)3}$.

Case (i): Replicated Latin Squares design of Type I
By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 36 & -4 & -4 \\ -4 & 36 & -4 \\ -4 & -4 & 36 \end{bmatrix}$$

b_1 = 4(109 + 121 + 98)+3x140 – 3 x 456 = 364
b_2 = 4(116 + 113 + 98)+3x140 – 3 x 456 = 360
b_3 = 4(105 + 109 + 98)+3x140 – 3 x 456 = 300

Since the missing values are of a particular pattern discussed in Subramani (1991), one can give explicit expressions for the estimates of the missing values as

$$X_i = \frac{\left[(p-1)(np+n-3)-(m-1)(p+n-3)\right]b_i + (p+n-3)\sum_{j=1}^{m} b_j}{\left[(p-1)(np+n-3)-(m-1)(p+n-3)\right]\left[(p-1)(np+n-3)+(p+n-3)\right]}$$

, i = 1,2,…..,m

It is given that m=3, n=3 and p=4. By substituting those values in the above equation the estimates of the missing values are obtained as:

$$X_1 = Y_{12(4)3} = \frac{\left[(4-1)(3*4+3-3)-(3-1)(4+3-3)\right]364 + (4+3-3)1024}{\left[(4-1)(4*3+3-3)-(3-1)(4+3-3)\right]\left[(4-1)(4*3+3-3)+(4+3-3)\right]} = 12.76$$

$$X_2 = Y_{23(4)3} = \frac{\left[(4-1)(3*4+3-3)-(3-1)(4+3-3)\right]360 + (4+3-3)1024}{\left[(4-1)(4*3+3-3)-(3-1)(4+3-3)\right]\left[(4-1)(4*3+3-3)+(4+3-3)\right]} = 12.66$$

$$X_3 = Y_{34(4)3} = \frac{\left[(4-1)(3*4+3-3)-(3-1)(4+3-3)\right]300 + (4+3-3)1024}{\left[(4-1)(4*3+3-3)-(3-1)(4+3-3)\right]\left[(4-1)(4*3+3-3)+(4+3-3)\right]} = 11.16$$

After substituting the estimated values of the missing values in their respective positions the treatment effects are obtained as:

$$T_1 = \frac{113}{12} - \frac{492.58}{48} = 9.417 - 10.262 = -0.845$$

$$T_2 = \frac{114}{12} - \frac{492.58}{48} = 9.500 - 10.262 = -0.762$$

$$T_3 = \frac{131}{12} - \frac{492.58}{48} = 10.917 - 10.262 = 0.655$$

$$T_4 = \frac{134.58}{12} - \frac{492.58}{48} = 11.215 - 10.262 = 0.953$$

The degrees of freedom for error component σ^2 is f* = 36-3 = 33

The estimate of the experimental error $\hat{\sigma}$ is obtained as

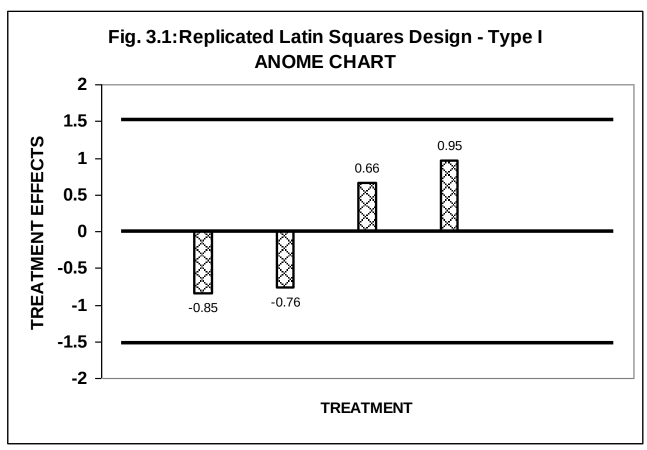
$$\hat{\sigma} = \sqrt{\left\{\left[5305.6 - \frac{60748.0}{12} - \frac{60889.5}{12} - \frac{61037.7}{12} - \frac{81180.4}{16} + \frac{3*242635.5}{48}\right]/33\right\}}$$

= 2.293585

The value of the critical factor is obtained as $h_{0.05}$ (4,33) = 2.64

The decision lines at α= 0.05 are obtained as
LDL = 0-2.293585x2.64x$\sqrt{3/48}$ = -1.5138
UDL = 0+2.293585x2.64x$\sqrt{3/48}$ = 1.5138

Now plot the treatment effects T_1, T_2, T_3 and T_4 as in Figure 3.1. From the Figure 3.1 it is observed that all the plotted treatment effects are within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.



Case (ii): Replicated Latin Squares design of Type II
By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 30 & -2 & -2 \\ -2 & 30 & -2 \\ -2 & -2 & 30 \end{bmatrix}$$

b_1 = 4(3x35 + 121 + 98) – 2 x 456 = 384
b_2 = 4(3x32 + 113 + 98) – 2 x 456 = 316
b_3 = 4(3x32 + 109 + 98) – 2 x 456 = 300

Since the missing values are of a particular pattern discussed in Subramani (1991), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{[(p-1)(np-2)-(m-1)(p-2)]b_i + (p-2)\sum_{j=1}^{m} b_j}{[(p-1)(np-2)-(m-1)(p-2)][(p-1)(np-2)+(p-2)]}$$

, i = 1,2,…..,m

It is given that m=3, n=3 and p=4. By substituting those values in the above equation the estimates of the missing values are obtained as:

$$X_1 = Y_{12(4)3} = \frac{[(4-1)(4*3-2)-(3-1)(4-2)]384 + (4-2)1000}{[(4-1)(4*3-2)-(3-1)(4-2)][(4-1)(4*3-2)+(4-2)]} = 14.40$$

$$X_2 = Y_{23(4)3} = \frac{[(4-1)(4*3-2)-(3-1)(4-2)]316 + (4-2)1000}{[(4-1)(4*3-2)-(3-1)(4-2)][(4-1)(4*3-2)+(4-2)]} = 12.28$$

$$X_3 = Y_{34(4)3} = \frac{[(4-1)(4*3-2)-(3-1)(4-2)]300 + (4-2)1000}{[(4-1)(4*3-2)-(3-1)(4-2)][(4-1)(4*3-2)+(4-2)]} = 11.78$$

After substituting the estimated values of the missing values in their respective positions the treatment effects are obtained as:

$$T_1 = \frac{113}{12} - \frac{494.46}{48} = 9.417 - 10.30125 = -0.885$$

$$T_2 = \frac{114}{12} - \frac{494.46}{48} = 9.500 - 10.30125 = -0.801$$

$$T_3 = \frac{131}{12} - \frac{494.46}{48} = 10.917 - 10.30125 = 0.615$$

$$T_4 = \frac{136.46}{12} - \frac{494.46}{48} = 11.215 - 10.30125 = 1.070$$

The degrees of freedom for error component σ^2 is f* = 30-3 = 27

The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left[5354.9268 - \frac{20586.7668}{4} - \frac{61385.0468}{12} - \frac{61547.3316}{12} + \frac{2*24}{48}\right]/27\right\}}$$

= 2.364728

The value of the critical factor is obtained as $h_{0.05}$ (4,27) = 2.68

The decision lines at α= 0.05 are obtained as

LDL = 0-2.68x2.364728x $\sqrt{3/48}$ = -1.5844
UDL =0 +2.68x2.364728x $\sqrt{3/48}$ = 1.5844

Now plot the treatment effects T_1, T_2, T_3 and T_4 as in Figure 3.2. From the Figure 3.2 it is observed that all the plotted treatment effects are within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.

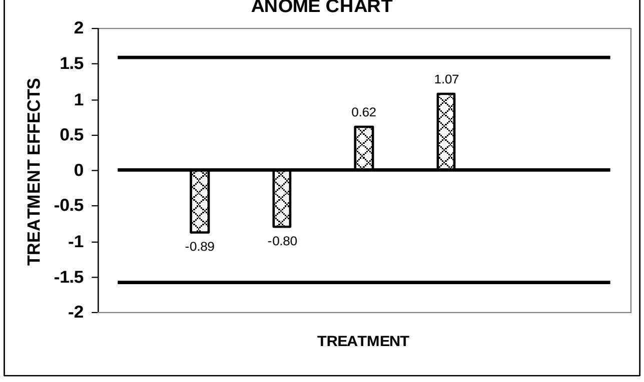


Fig. 3.2:Replicated Latin Squares Design - Type II ANOME CHART

Case (iii): Replicated Latin Squares design of Type III
By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{bmatrix}$$

b_1 = 4(3x35 + 3x31 + 98) - 3x140 - 456 = 308
b_2 = 4(3x32 + 3x38 + 98) – 3x140 - 456 = 356
b_3 = 4(3x32 + 3x31 + 98) – 3x140 - 456 = 272

Since the missing values are of a particular pattern discussed in Subramani (1991), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{[(p-1)(np-n-1)-(m-1)(p-n-1)]b_i + (p-n-1)\sum_{j=1}^{m} b_j}{[(p-1)(np-n-1)-(m-1)(p-n-1)][(p-1)(np-n-1)+(p-n-1)]}$$

, i = 1,2,…..,m

When $p-n-1=0$, the above equation is reduced to

$$X_i = \frac{b_i}{(p-1)(np-n-1)}, \quad i=1, 2, \ldots, m.$$

It is given that m=3, n=3 and p=4. By substituting those values in the above equation the estimates of the missing values are obtained as:

$$X_1 = Y_{12(4)3} = \frac{308}{(4-1)(4*3-3-1)} = 12.83$$

$$X_2 = Y_{23(4)3} = \frac{356}{(4-1)(4*3-3-1)} = 14.83$$

$$X_3 = Y_{34(4)3} = \frac{272}{(4-1)(4*3-3-1)} = 11.33$$

After substituting the estimated values of the missing values in their respective positions the treatment effects are obtained as:

$$T_1 = \frac{113}{12} - \frac{494.99}{48} = 9.417 - 10.3123 = -0.896$$

$$T_2 = \frac{114}{12} - \frac{494.99}{48} = 9.500 - 10.3123 = -0.812$$

$$T_3 = \frac{131}{12} - \frac{494.99}{48} = 10.917 - 10.3123 = 0.604$$

$$T_4 = \frac{136.99}{12} - \frac{494.99}{48} = 11.416 - 10.3123 = 1.104$$

The degrees of freedom for error component σ^2 is f* = 24-3 = 21

The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left[5370.9067 - \frac{20627.2467}{4} - \frac{20835.9067}{4} - \frac{61692.2601}{12} + \frac{82037.4201}{16} + \frac{245015.1001}{48}\right] / 21\right\}}$$

= 2.13716

The value of the critical factor is obtained as $h_{0..05}$ (4, 21) = 2.74

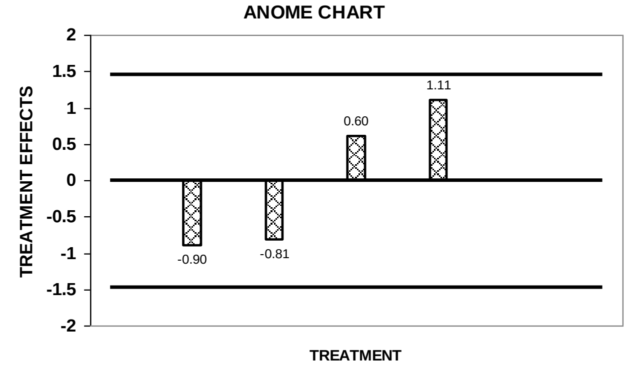
The decision lines at α= 0.05 are obtained as

LDL = 0 - 2.13716x2.74x$\sqrt{3/48}$ = -1.464

UDL = 0 + 2.13716x2.74x$\sqrt{3/48}$ = 1.464

Now plot the treatment effects T_1, T_2, T_3 and T_4 as in Figure 3.3. From the Figure 3.3 it is observed that all the plotted treatment effects are within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.

Fig. 3.3: Replicated Latin Squares Design - Type III ANOME CHART



4. F- Square Designs

In this section, the step by step procedure of analyzing missing data from F- Square design is presented and also illustrated with the help of a numerical example. The procedure is discussed in Section 4.1., where as the numerical example is given in Section 4.2.

4.1. ANOME to Analyze Missing Data from F- Square Designs

Consider a F-Square design with t treatments in a pxp square such that each treatment appears λ times in each row and in each column. For want of space, the definition, design and applications of F-Square designs are not discussed here and the readers are referred to Hedayat and Seiden (1970), Hedayat, Raghavarao and Seiden (1975), Subramani and Aggarwal (1993) and the references cited therein. Let m be the number of missing values then $m < (p-1)(p-2)+(p-t)$. The proposed ANOME procedure is given below:

Step 1: The model of a F-Square design is

$$Y_{ij(k)} = \mu + r_i + c_j + t_k + e_{ij(k)}, \quad i, j = 1,2,\ldots, p \; and \; k = 1,2,\ldots,t$$

where $Y_{ij(k)}$ is the observation from i^{th} row, j^{th} column and k^{th} treatment; r_i is the effect of i^{th} row; c_j is the effect of j^{th} column; t_k is the effect of k^{th} treatment and $e_{ij(k)}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani and Aggarwal (1993) as

$$A = (a_{ij}) = \begin{cases} (p-1)(p-2)+(p-t) & \text{if } i = j \\ -(p-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular row or column but from different treatments} \\ -(t-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular treatment but from different rows or columns} \\ -(p+t-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular row and of a particular treatment or in a particular column and of a particular treatment} \\ 2 & \text{Otherwise} \end{cases}$$

Step 3 : The elements of the vector b are obtained as

$b_i = p(R_{(i)} + C_{(i)} + T_{(i)} / \lambda) - 2G'$, i= 1, 2, …, m.

where $R_{(i)}$, $C_{(i)}$, and $T_{(i)}$ are respectively the row, column and treatment totals corresponding to the i^{th} missing value and G' is the grand total of all known observations

Step 4 : The estimates of the missing values are obtained as ***x = A^-1^ b***

Step 5 : Substitute the estimates of the missing values in their respective positions and then obtain the different treatment effects are obtained as

$T_k = \bar{Y}_{..(k)} - \bar{Y}$, k=1, 2, …, t.

where $\bar{Y}_{..(k)}$ is the mean value of the k^{th} treatment and $\bar{Y}$ is the grand mean.

Step 6: Determine the degrees of freedom for error sum of squares as f* =f-m, where $f = (p-1)(p-2) + (p-t)$

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(k)}^2 - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{\lambda p}\sum_{k=1}^{t} T_k^2 + \frac{2}{p^2}G^2\right) / f^*\right\}}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$0 \pm \sigma h_\alpha \sqrt{(p-1)/p^2}$,where

$h_\alpha = h_\alpha(p, f^*)$

Step 9: Plot the treatment effects $T_1, T_2, \ldots, T_t$ against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous at α level of significance.

4.2. Numerical Example

Consider the F-Square design with 3 treatments in 6 rows and 6 columns such that each treatment is replicated 2 (= λ) times in each row and in each column. The design is obtained from the latin square design given in John and Quenouille (1977, page 55) by replacing the treatments D, E and F respectively as A, B and C. The resulting data is given in the Table 4.1.

Table 4.1.: Missing Data in F-Square Design

Rows	Columns						Total
	1	2	3	4	5	6	
1	B 42.6	A ***	C 28.5	C 32.2	B 32.2	A 25.7	161.2
2	C 41.5	B 40.2	B 34.4	C 32.6	A 33.9	A ***	182.6
3	C 32.2	C 33.3	A 25.7	B 32.6	A ***	B 30.2	154.0
4	B 35.0	C 39.1	A ***	A 35.0	B 34.4	C 35.6	179.1
5	A 33.9	B 36.1	B 35.0	A ***	C 40.8	C 41.9	187.7
6	A ***	A 28.5	C 31.6	B 23.3	C 32.6	B 39.8	155.8
Total	185.2	177.2	155.2	155.7	173.9	173.2	1020.4

Let X_1, X_2, X_3, X_4, X_5 and X_6 be the corresponding estimated values of the missing values $Y_{12(1)}$, $Y_{26(1)}$, $Y_{35(1)}$, $Y_{43(1)}$, $Y_{54(1)}$ and $Y_{61(1)}$. By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 23 & -1 & -1 & -1 & -1 & -1 \\ -1 & 23 & -1 & -1 & -1 & -1 \\ -1 & -1 & 23 & -1 & -1 & -1 \\ -1 & -1 & -1 & 23 & -1 & -1 \\ -1 & -1 & -1 & -1 & 23 & -1 \\ -1 & -1 & -1 & -1 & -1 & 23 \end{bmatrix}$$

b_1 = 6(161.2 +177.2 + 182.7/2) – 2x1020.4 = 537.7
b_2 = 6(182.6 +173.2 + 182.7/2) – 2x1020.4 = 642.1
b_3 = 6(154.0 +173.9 + 182.7/2) – 2x1020.4 = 474.7
b_4 = 6(179.1 +155.2 + 182.7/2) – 2x1020.4 = 513.1
b_5 = 6(187.7 +155.7 + 182.7/2) – 2x1020.4 = 567.3
b_6 = 6(155.8 +185.2 + 182.7/2) – 2x1020.4 = 553.3

Since the missing values are of a particular pattern discussed in Subramani and Aggarwal (1993), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{[p(p-2)/(t-2)-m]b_i + \sum_{j=1}^{m} b_j}{p(p-2)[p(p-2)/(t-2)-m]},$$

i = 1, 2, …., m.

It is given that m=6, t=3 and p=6. By substituting those values in the above equation, the estimates of the missing values are obtained as:

$$X_1 = Y_{12(1)} = \frac{[6(6-2)/(3-2)-6]537.7 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 30.0167$$

$$X_2 = Y_{26(1)} = \frac{[6(6-2)/(3-2)-6]642.1 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 34.3667$$

$$X_3 = Y_{35(1)} = \frac{[6(6-2)/(3-2)-6]474.7 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 27.3917$$

$$X_4 = Y_{43(1)} = \frac{[6(6-2)/(3-2)-6]513.1 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 28.9917$$

$$X_5 = Y_{54(1)} = \frac{[6(6-2)/(3-2)-6]567.7 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 31.2667$$

$$X_6 = Y_{61(1)} = \frac{[6(6-2)/(3-2)-6]553.3 + 3288.6}{6(6-2)[6(6-2)/(3-2)-6]} = 30.6667$$

After substituting the estimated values of the missing values in their respective positions the treatment effects are obtained as:

$$T_1 = \frac{365.4}{12} - \frac{1203.1}{36} = 30.4500 - 33.4195 = -2.9694$$

$$T_2 = \frac{415.8}{12} - \frac{1203.1}{36} = 34.6500 - 33.4195 = 1.2306$$

$$T_3 = \frac{421.9}{12} - \frac{1203.1}{36} = 35.1583 - 33.4195 = 1.7389$$

The degrees of freedom for error component σ^2 is f* = 23-6 = 17. The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(k)}^2 - \frac{1}{p}\sum_{i=1}^{p} R_i^2 - \frac{1}{p}\sum_{j=1}^{p} C_j^2 - \frac{1}{\lambda p}\sum_{k=1}^{t} T_k^2 + \frac{2}{p^2}G^2\right)/ f^*\right\}}$$

$$\hat{\sigma} = \sqrt{\left\{\left(40994.4 - \frac{242559.7}{6} - \frac{242022.6}{6} - \frac{484406.5}{12} + \frac{2*1447450}{36}\right)/17\right\}}$$

$$\hat{\sigma} = 4.0397$$

The value of the critical factor is obtained as $h_{0.05}$ (3, 17) = 2.65

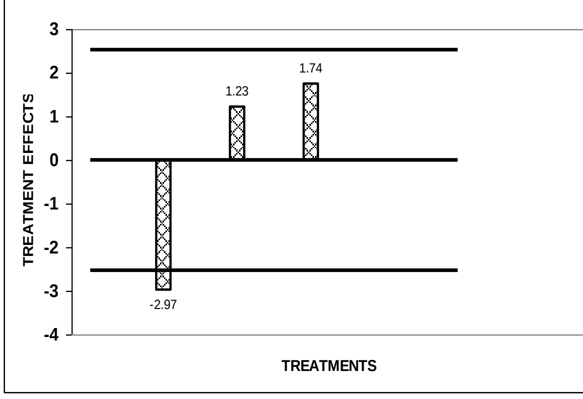
The decision lines at α= 0.05 are obtained as

LDL = 0-4.0397x2.65 x $\sqrt{2/36}$ = - 2.5232

UDL = 0+4.0397x2.65 x $\sqrt{2/36}$ = 2.5232

Now plot the treatment effects T_1, T_2 and T_3 as in Figure 4.1. From the Figure 4.1 it is observed that the plotted treatment effect T_1 falls outside the decision line and hence one can conclude that the effects of different treatments are not the same at 5% level of significance.

Fig. 4.1: F-SQUARE DESIGN - ANOME CHART



5. Cross over Designs Without Residual Effect

In this section, the step by step ANOME procedure of analyzing missing data from Cross-over designs without residual effect is presented and also illustrated with the help of a numerical example. The proposed ANOME procedure is discussed in Section 5.1., where as the numerical example is given in Section 5.2.

5.1. ANOME to Analyze Missing Data from Cross-over Designs without Residual Effect

Consider a Cross-over design with t treatments in t rows and in p columns. For want of space, we are not discussing the definition, design and applications of Cross-over designs and the readers are referred to Cochran and Cox (1957) and Subramani (1994) and the references cited there in. Let m be the number of missing values then m<(t-1)(p-2). The proposed ANOME procedure is given below:

Step 1: The model of the Cross-over design is

$$Y_{ij(k)} = \mu + r_i + c_j + t_k + e_{ij(k)}, \qquad i,k = 1,2,...,t \text{ and } j = 1,2,...,p$$

where $Y_{ij(k)}$ is the observation from ith row, jth column and k^{th} treatment; r_i is the effect of ith row; c_j is the effect of jth column; t_k is the effect of k^{th} treatment and $e_{ij(k)}$ is the error component with mean 0 and variance σ^2.

Step 2 : The elements of the matrix A are obtained from Subramani (1994) as

$$A = (a_{ij}) = \begin{cases} (t-1)(p-2) & \text{if } i = j \\ -(p-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular column} \\ -(t-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are from a particular treatment (row) but from different rows (treatments) different rows} \\ -(2t-2) & \text{if } i^{th} \text{ and } j^{th} \text{ missing values are of a particular row and of a particular treatment} \\ 2 & \text{Otherwise} \end{cases}$$

Step 3 : The elements of the vector b are obtained as

$$b_i = tR_{(i)} + pC_{(i)} + tT_{(i)} - 2G'$$

where $R_{(i)}$, $C_{(i)}$ and $T_{(i)}$ are respectively the row, column and treatment corresponding to the ith missing value and G' is the grand total of all known observations.

Step 4 : The estimates of the missing values are obtained as ***x = A^{-1} b***

Step 5 : Substitute the estimates of the missing values in their respective positions and then obtain different treatment effects. The kth treatment effect is obtained as

$$T_k = \overline{Y}_k - \overline{Y}, \quad k=1, 2, \ldots, t$$

where $\overline{Y}_k$ is the mean value of the kth treatment and $\overline{Y}$ is the grand mean

Step 6: Determine the degrees of freedom for error sum of squares as f* =f-m, where $f = (t-1)(p-2)$

Step 7: The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{t}\sum_{j=1}^{p} y_{ij(k)}^2 - \frac{1}{p}\sum_{i=1}^{t} R_i^2 - \frac{1}{t}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{k=1}^{t} T_k^2 + 2G^2\right) / f^*\right\}}$$

Step 8: Compute the decision lines (LDL and UDL) for the desired risk α as follows:

$$0 \pm \sigma h_\alpha \sqrt{(t-1)/tp},$$

where $h_\alpha = h_\alpha(t, f^*)$

Step 9: Plot the treatment effects $T_1, T_2, \ldots, T_t$ against the decision lines and draw the conclusion that the treatment effects are significant if at least one of the plotted points falls outside the decision lines. Otherwise conclude that the treatment effects are homogeneous at α level of significance.

5.2. Numerical Example

Consider the Cross-over design with 2 treatments, 2 rows and 10 columns given in Cochran and Cox (1957, page 130). That is t=2 and p=10. In the data, assume that the observations in the cells (1,4), (1,5) and (1,7) are missing, which leads to the observations $Y_{14(2)}$, $Y_{15(2)}$ and $Y_{17(2)}$ are the missing values. The resulting data are given in Table 5.1 below:

Table 5.1.: Missing Data in Cross-over Designs

Rows	Columns										Total
	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	
R1	A	B	A	B	B	A	B	A	A	B	161
	30	21	22	***	***	29	***	12	23	24	
R2	B	A	B	A	A	B	A	B	B	A	158
	14	21	5	22	18	17	16	14	8	23	
Total	44	42	27	22	18	46	16	26	31	47	319

Let X_1, X_2 and X_3 be the corresponding estimated values of the missing values $Y_{14(2)}$, $Y_{15(2)}$ and $Y_{17(2)}$. By using the proposed method the matrix A and elements of the vector b are obtained as given below:

$$A = \begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix}$$

b_1 = 2*161 +10x22+ 2x103 – 2 x 319 = 110
b_2 = 2*161 +10x18+ 2x103 – 2 x 319 = 70
b_3 = 2*161 +10x16+ 2x103 – 2 x 319 = 50

Since the missing values are of a particular pattern discussed in Subramani (1994), one can give explicit expressions for the estimates of the missing values as:

$$X_i = \frac{(p-2m)b_i + 2\sum_{j=1}^{m} b_j}{p(t-1))(p-2m)},$$

i = 1, 2, …., m.

It is given that t=2, m=3 and p=10. By substituting those values in the above equation, the estimates of the missing values are obtained as:

$$X_1 = Y_{14(2)} = \frac{(10-6)x110 + 2x230}{10(2-1)(10-2x3)} = \frac{900}{40} = 22.5$$

$$X_2 = Y_{15(2)} = \frac{(10-6)x110 + 2x230}{10(2-1)(10-2x3)} = \frac{900}{40} = 22.5$$

$$X_3 = Y_{17(2)} = \frac{(10-6)x50 + 2x230}{10(2-1)(10-2x3)} = \frac{660}{40} = 16.5$$

After substituting the estimated values of the missing values in the respective positions the treatment effects are obtained as:

$$T_1 = \frac{216}{10} - \frac{376.5}{20} = 21.6 - 18.825 = 2.775$$

$$T_2 = \frac{160.5}{10} - \frac{376.5}{20} = 216.05 - 18.825 = -2.775$$

The degrees of freedom for error component σ^2 is f* = 8-3 = 5

The estimate of the experimental error $\hat{\sigma}$ is obtained as

$$\hat{\sigma} = \sqrt{\left\{\left(\sum_{i=1}^{t}\sum_{j=1}^{p} y_{ij(k)}^2 - \frac{1}{p}\sum_{i=1}^{t} R_i^2 - \frac{1}{t}\sum_{j=1}^{p} C_j^2 - \frac{1}{p}\sum_{k=1}^{t} T_k^2 + 2G^2\right) / f^*\right\}}$$

$$\hat{\sigma} = \sqrt{\left\{\left(7839.75 - \frac{72706.25}{10} - \frac{14759.75}{2} - \frac{72416.25}{10} + \frac{2*141752.25}{20}\right) / 5\right\}}$$

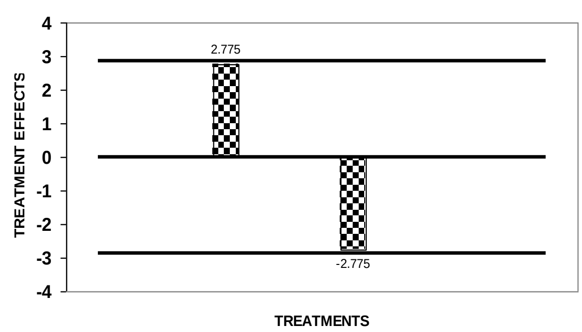
$\hat{\sigma}$ = 4.9568

The value of the critical factor is obtained as
$h_{0.05}$ (2,5) = 2.57

The decision lines at α= 0.05 are obtained as
LDL = 0-4.9568x2.57 x $\sqrt{1/20}$ = - 2.8485
UDL = 0+4.9568x2.57 x $\sqrt{1/20}$ = + 2.8485

Now plot the treatment effects T_1 and T_2 as in Figure 5.1. From the Figure 5.1 it is observed that all the plotted treatment effects fall within the two decision lines and hence one can conclude that the effects of different treatments are the same at 5% level of significance.

Fig. 5.1: Cross- over Design without Residual Effect - ANOME CHART



Summary

It is well known that the ANOME procedure is useful to assess the Engineering significance as well as statistical significance from the experimental data with factors at fixed levels. However, the drawback of this procedure is that it is used so far only for the balanced experimental designs. That is, the ANOME Procedure is applicable only if we have equal number of observations in each cell of the experimental designs. In this series Part I and in Part II, we have presented a step-by-step method for the use of ANOME procedure to analyze the missing data from Latin Square Designs, Graeco Latin Square Designs, Hyper Graeco Latin Squares Designs, Replicated Latin Squares Designs of Type I, Type II and Type III, F-Square Designs and Cross-over Designs without Residual Effects. The proposed ANOME procedure is also illustrated with the help of numerical examples. The key point in using the ANOME procedure to analyze the missing data is to get the complete data by inserting estimates of the missing values in their respective cells. For estimating several missing values from Latin Square Designs, Graeco-Latin Square Designs, Hyper-Graeco- Latin Squares Designs, Cross-over Designs and F-Square Designs one may refer to Subramani and Ponnuswamy (1989), Subramani (1991, 93, 94, 2008b) and Subramani and Aggarwal (1993).

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