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PREDICTION OF EFFECTIVE TRANSVERSE THERMAL CONDUCTIVITY OF BORON FIBER-REINFORCED COMPOSITES

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ABSTRACT

Applicability of the finite element model (FEM) in predicting the effective transverse thermal conductivity (k_2) of the unidirectional fiber reinforced composites is systematically studied. A 3-D finite element model for the array of square unit cell of long circular cylinder with appropriate thermal boundary conditions is developed. FEM software ANSYS 10.0 is successfully executed and the effective transverse thermal conductivity is evaluated for various fiber volume fractions (V_f). The results are validated with the experimental and analytical results available in the literature and found to be quite coherent. The developed single model is found suitable in accurately predicting k_2 at all values of V_f in the range and thus eliminate the complexity of developing different analytical expressions for different range of V_f. The methods such as analytical and 2-D FEM developed so far fails in accommodating fiber anisotropy, imperfections in matrix, fiber and fiber-matrix interface. The present model is capable of predicting k_2 in all the above cases. Finally transverse effective thermal conductivity is evaluated for Boron composites with a range of matrix thermal conductivity values.

Keywords: FEM, Transverse conductivity, Composites, Unit cell

1. INTRODUCTION

Prediction of thermo-physical properties of composite material is of great concern as these materials are widely used in aero-space applications. Considerable research is being carried out in this area using analytical, experimental and numerical models. Much effort, time and expense would be saved with the use of latest available software as modeling and analysis becomes much simpler and flexible.

The effective microscopic thermal properties of a composite will in general be dependent on the individual properties of the constituents, their relative volume fractions and their micro-structural arrangement. When the reinforcement consists of aligned long continuous fibers, then the effective properties become anisotropic. The fiber direction effective conductivity (k_1) of each lamina is satisfactorily predicted by a simple rule of mixtures and is independent of geometrical arrangement, since both fiber and matrix present continuous path for the conduction of heat.

In transverse direction, due to geometry and orientation of fibers, heat transfer path may not be continuous and prediction of effective transverse thermal conductivity (k_2) is not straight forward. Although most composites possesses fibers of random distributions, great insight of microstructure on the effective properties can be gained from investigation of composites with periodic structures, thus it is sufficient to draw conclusions for the whole system considering only unit cell.

Analysis of periodic systems date back to the work of Rayleigh [1] who considered the effective electrical conductivity of a composite material with equal sized spherical inclusions arranged in a simple cubic array. The literature dealing with theoretical prediction of the transverse thermal conductivity of composite materials is considerable and several reviews have been published [2-5]. Perrins [6] extended Rayleigh's work [1] to enable the calculation of the transport properties of circular cylinders in square and hexagonal arrays of unit cells. Grove [7] computed transverse thermal conductivity in continuous unidirectional fiber composite materials using both 2-D FEM and statistical techniques for a range of fiber volume fractions up to 0.5 and k_f/k_m from 2 to 500. Yuan Lu [8] used boundary collocation scheme for calculation of transverse effective thermal conductivity of

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2-dimensional periodic arrays of long circular and square cylinders with square array and long circular cylinders with hexagonal array for a complete range of fiber volume fractions and for k_f/k_m between 0 to ∞. Islam [9] used 2-dimensional FEM to predict the transverse thermal conductivity of both square and circular cross section fibers for perfect bonding at fiber-matrix interface as well as with interfacial barrier by using four different sets of thermal boundary conditions. Tai [10] proposed two circular filaments in a unit cell of square packing array and obtained the transverse thermal conductivity of unidirectional fibers.

In this paper, a 3-dimensional finite element model for the array of square unit cell of long circular cylinders is developed with appropriate thermal boundary conditions at all faces. The model is tested for entire range of fiber volume fractions and k_f/k_m of 0 to ∞. Output is compared with experimental and analytical results presented by various researchers.

2. FINITE ELEMENT MODEL

A schematic diagram of the unidirectional fiber composite is shown in Fig. 1, where the fibers are arranged in a square array. A Representative Volume Element (R.V.E.) in the form of a square unit cell is adopted for the present analysis. The cross-sectional area of fiber relative to the total cross-sectional area of the unit cell (Fig. 2) is a measure of the volume of fiber relative to the total volume of the composite. This fraction is an important parameter in composite materials and is called fiber volume fraction (V_f).

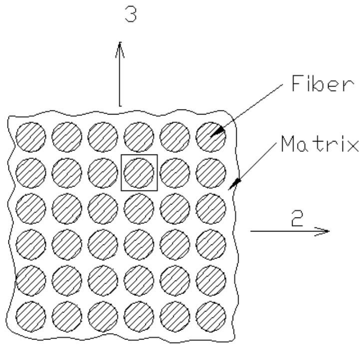

Fig. 1. Concept of unit cells

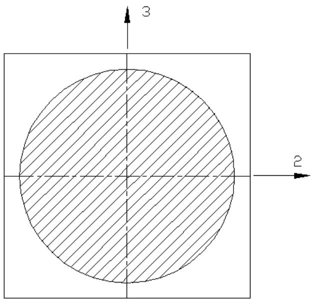

Fig. 2. Isolated unit cell of Square packed array

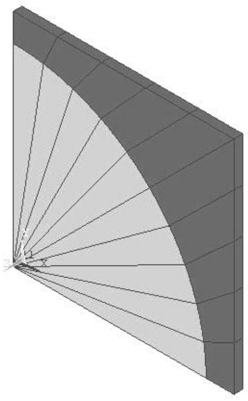

Fig. 3. One-forth model

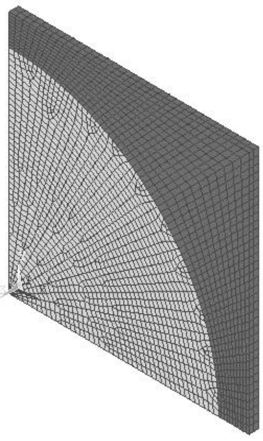

Fig. 4. One-forth model with mesh

The 1-2-3 coordinate system shown in Fig. 2 is used to study the behavior of a unit cell (The direction 1 is along the fiber axis and normal to the plane of the 2D figure shown). The isolated unit cell behaves as a part of a larger array of unit cells.

It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to 1-2-3 coordinate system. Therefore, a one forth portion of the unit cell is modeled (Fig. 3) for the prediction of thermal properties. The 3D Finite Element mesh on one forth portion of the unit cell is shown in Fig. 4.

Element Type

The element SOLID90 of ANSYS 10.0 has 20 nodes with a single degree of freedom, temperature, at each node. The 20-node elements have compatible temperature shapes and are well suited to model curved boundaries.

Materials

The Boron fiber - Polymer matrix materials with following properties are used.

Boron Fiber: $k_f = 38$ W/m-K

Polymer Matrix: $k_m =$ 0.1 to 1.0 W/m-K

Boundary Conditions: Temperature boundary conditions for one-fourth model are as follows. Sides of the unit cell is taken as '2a' and length is taken sufficiently long.

$$T(x, 0) = T_1; \qquad T(x, a) = T_2$$

All other faces are subjected to adiabatic boundary conditions.

3. RESULTS AND DISSCUSSION

The effective transverse thermal conductivity is calculated using the equation

$$q_y = -k_2 \frac{\partial T}{\partial y}$$

Heat flux and the temperature gradient in the above equation are obtained from the finite element solution. Figures 5 & 6 show the comparison of the results with Springer and Tsai [11] and Thornburg and Pears [12] at k_f/k_m values of 666 and 4.4. In both the cases the proposed model is predicting higher k_2 than [11]. However at lower fiber volume fractions, reasonably close agreement is observed. In the case of k_f/k_m = 666, experimental values [12] are higher than the proposed model. This may be due to random distribution of fibers where some of the fibers may be packed very closely, touching each other in actual model which lead to higher effective transverse thermal conductivity.

In Table 1 & 2 the present model is compared with analytical solutions of [6] and [8] for k_f/k_m values of 5,10,50, and ∞ at fiber volume fractions between 0.1 and 0.785. It has been observed that an excellent agreement between present model and [6] and [8] for all values of k_f/k_m and for volume fractions from 0.1 to 0.78. It is also observed that the present model is able to predict k_2 at a volume fraction of 0.785 where the analytical models failed.

The present model is extended to determine the effective transverse thermal conductivity of Boron fiber and thermosetting plastic matrix. Table 3 shows the effective thermal conductivity of Boron fiber and thermosetting plastic matrix composites for rage of matrix conductivities between 0.1 – 1.0 w/m-k. For higher values of k_f/k_m a steep raise in k_2 is observed at volume fractions beyond 0.65 and a gradual raise in k_2 is found with increase in V_f as k_f/k_m decreases. At 60% fiber volume fraction which is normally employed by industry, k_2 is observed to be between 1.1 - 10% of Boron fiber conductivity for the range of matrix conductivities 0.1-1.0 W/m-k

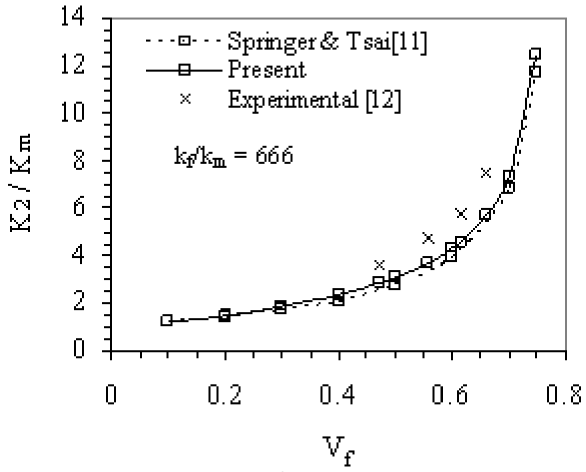

Fig. 5. Comparison of the present work with Experimental and analytical results at k_f/k_m = 666

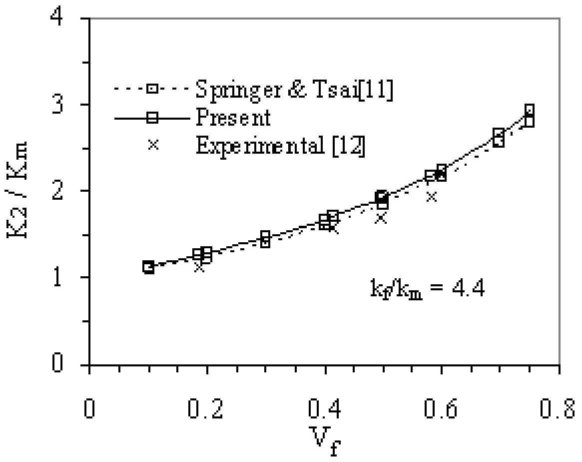

Fig. 6. Comparison of the present work with experimental and analytical results at k_f/k_m = 4.4

Table 1. Comparison of present model with analytical results at $k_f/k_m = 5, 10$

	$k_f/k_m = 5$			$k_f/k_m = 10$	
V_f	Ref[6]	Ref[8]	Present	Ref[6]	Present
0.1	1.1429	1.14286	1.143	1.1782	1.178
0.2	1.3078	1.30777	1.308	1.3915	1.391
0.3	1.5007	1.50069	1.501	1.6520	1.652
0.4	1.7307	1.73075	1.731	1.9806	1.981
0.5	2.0130	2.01298	2.013	2.4155	2.4155
0.6	2.3744	2.37438	2.374	3.0372	3.037
0.7	2.8709	2.87089	2.871	4.0617	4.062
0.75	3.2099	3.20990	3.210	4.9443	4.944
0.76	3.2900	3.28959	3.290	5.1867	5.189
0.77	3.3747	3.37468	3.375	5.4674	5.468
0.78	3.4663	3.46627	3.466	5.8037	5.804
0.785	3.5150	3.51500	3.515	6.004	6.01

Table 2. Comparison of present model with analytical results at $k_f/k_m = 50, \infty$

	$k_f/k_m = 50$			$k_f/k_m = \infty$		
V_f	Ref[6]	Ref[8]	Present	Ref[6]	Ref[8]	Present
0.1	1.2126	1.21259	1.213	1.2222	1.22223	1.222
0.2	1.4760	1.47599	1.476	1.5003	1.50031	1.500
0.3	1.8125	1.81253	1.813	1.8602	1.86019	1.860
0.4	2.2633	2.26327	2.263	2.3510	2.35100	2.351
0.5	2.9146	2.91459	2.9147	3.0802	3.08020	3.080
0.6	3.9881	3.98815	3.988	4.3418	4.34182	4.342
0.7	6.3359	6.33592	6.336	7.4327	7.43271	7.433
0.75	9.5355	9.53551	9.536	12.7511	12.7511	12.752
0.76	10.8187	10.8187	10.829	15.4412	15.4412	15.463
0.77	12.7411	12.7412	12.745	20.4334	20.4334	20.441
0.78	16.310	16.3104	16.312	35.934	35.9342	35.945
0.785	20.5	20.5	20.492	-	-	137.608

Table 3. Effective Transverse Thermal Conductivity of Boron fiber composite

V_f	$k_m = 0.1$	$k_m = 0.2$	$k_m = 0.3$	$k_m = 0.4$	$k_m = 0.5$	$k_m = 0.6$	$k_m = 0.7$	$k_m = 0.8$	$k_m = 0.9$	$k_m = 1.0$
0.1	0.1221	0.244	0.366	0.487	0.608	0.729	0.845	0.97	1.09	1.21
0.2	0.15	0.299	0.447	0.595	0.742	0.889	1.034	1.18	1.325	1.469
0.3	0.185	0.369	0.552	0.734	0.914	1.093	1.271	1.448	1.624	1.798
0.4	0.234	0.465	0.694	0.921	1.146	1.3686	1.5889	1.8071	2.023	2.237
0.5	0.306	0.607	0.903	1.196	1.484	1.769	2.049	2.325	2.598	2.867
0.6	0.4292	0.8482	1.2583	1.659	2.0509	2.4347	2.8097	3.177	3.5368	3.8892
0.65	0.5368	1.0571	1.5624	2.0527	2.5291	2.992	3.4421	3.8799	4.3062	4.7213
0.7	0.7267	1.4212	2.0863	2.7236	3.335	3.9223	4.4869	5.0302	5.5534	6.0579
0.75	1.2202	2.3398	3.3715	4.326	5.2121	6.0374	6.8086	7.5311	8.2099	8.8492
0.76	1.4623	2.7746	3.96	5.0377	6.0223	6.9263	7.7601	8.5321	9.2497	9.9189
0.77	1.889	3.516	4.933	6.182	7.292	8.288	9.187	10.004	10.752	11.439
0.78	3.0813	5.404	7.229	8.712	9.947	10.998	11.907	12.705	13.413	14.049
0.785	7.357	10.224	11.887	13.038	13.92	14.64	15.247	15.779	16.252	16.68

4. CONCLUSION

A single 3-dimensional finite element model is developed using square array of circular cross section fibers to predict the effective transverse thermal conductivity most accurately at all values of fiber volume fractions. The model is extended to determine the k_2 of composite with Boron fiber. Transverse thermal conductivity is presented in the form of table is useful to the designers in selection of matrix material and fiber volume fraction to meet the requirement. The same model can be extended to predict effective thermal conductivities of the composite with imperfections such as voids in matrix, cracks in fiber as well as matrix and fiber-matrix debonding.

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