THERMO - ELASTIC ANALYSIS OF THICK SKEW LAMINATED COMPOSITE PLATE WITH ELLIPTICAL CUTOUT

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ABSTRACT

The behavior of a cross-ply laminated composite skew plate with elliptical hole subjected to temperature loading has been investigated in the present analysis. A finite element method, which works on the basis of three-dimensional theory of elasticity, is employed to evaluate the transverse deflection, in-plane stresses and interlaminar stresses. The results obtained by varying the skew angle, size of the cutout and the effect of elliptical cutout orientation are discussed. The magnitudes of the in-plane stresses for temperature loading are observed to be less at higher skew angles and for larger size of the cutout. There is no significant Variation of the transverse deflection. It is also observed that the configuration 3 of the ellipse (Major axis of the ellipse Collinear with the longer diagonal of the skew plate) is found to be better with respect to the major stresses point of view. The solutions of skew structures considered in the present analysis will be useful for the construction of safe and efficient structures like skew bridges and swept wings of aircraft structures.

Key words*:* FEM, Skew Laminate, Cutout, Interlaminar stresses

1. INTRODUCTION

The increasing use of fibre reinforced laminates in space vehicles; aircrafts, automobiles, ships and chemical vessels have necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy and non-homogeneity and larger ratio of longitudinal to transverse modulii of these new materials demand improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like (i) for the passage of various cables, (ii) for undertaking maintenance work and (iii) for fitting auxiliary equipment. Depending upon the nature of application, these structural elements are acted upon by mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending, and shear deformation modes. To capture the full mechanical behavior, it must be described by three dimensional elasticity theories.

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches have been proposed. Srinivas and Rao[1] and Srinivas *et al.*[2] presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. Zhang and Zhang[3] presented a new concise procedure for obtaining the static exact solution of composite laminates with piezo-thermo-elastic layers under cylindrical bending using the basic coupled thermo-electro-elastic differential equations. Setoodeh and Karami[4] employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Pagano *et al.*[5] has given exact solutions for the deflections and stresses of a cross- ply laminated rectangular composites without holes using elasticity theory.

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Kong and Cheung[6] proposed a displacement-based, three-dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a three-dimensional inhomogeneous anisotropic elastic body. Prasad and Shuart[7] presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Ukadgaonker *et al.*[8] gave a general solution for bending of symmetric laminates with holes. Morley *et al.*[9] developed an elementary bending theory for the small displacements of initially flat isotropic skew plates without hole. Karami *et al.*[10] has applied Differential Quadrature Method (DQM) for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates without hole. From the review of available literature it is observed that the static analysis of skew plates with cutouts using elasticity theory has not been studied. The behavior of a laminate with skew edges and having various types of cutouts is different from the one without skew edges and/or cutouts. So it is necessary to analyze this kind of problem using elasticity theory based finite element method to evaluate for the most accurate behavior of thick laminated skew plates with cutouts.

1.1 Skew Laminate

The term ‘skew’ in skew laminate refers to oblique, swept or parallelogram. In case of skew plate the angle between the adjacent sides of the plate is not equal to 90^0. If opposite sides of the plate are parallel, it becomes a parallelogram and when their lengths are equal, the plate is called a rhombic plate. In the present analysis a rhombic laminated plate is considered by varying the skew angle from 0^0 to 50^0 as shown in Fig.1.

2. PROBLEM STATEMENT

The present work aims at filling of the knowledge gaps in the existing literature. The research problem deals with the Thermo elastic analysis of thick skew laminated cross-ply graphite epoxy composite plate with elliptical cutout by elasticity theory based on finite element method.

3. PROBLEM MODELING

3.1Geometric Modeling

The Fig.1 shows the in-plane dimensions of the laminate considered for the present analysis. The dimensions for ‘*l*’ and ‘*b*’ are taken as 20mm where ‘*l*’ is the length of the horizontal side and ‘*b*’ is the length of the inclined side of the skew laminate. ‘*d*’ is the length of the minor axis of the ellipse. Major axis of the ellipse is taken as twice the length of the minor axis.

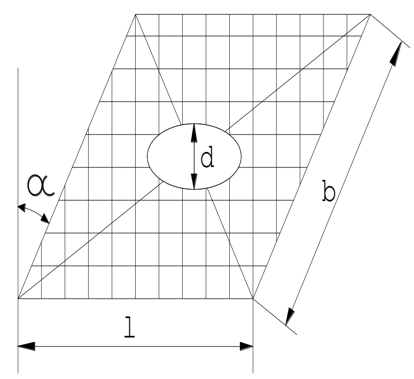

Fig 1. Skew laminated composite plate with elliptical cutout.

The value of *d* is determined from the ratio of *d/l* which is varied from 0.1 to 0.4, and the skew angle α is varied from 0^0 to 50^0, the thickness of the plate is fixed from the length to thickness ratio *l/h* (*s*=10). The individual layers are arranged so that the total thickness of the layers oriented in x- direction ($\theta = 0^0$) is equal to the total thickness of the layers oriented in y- direction ($\theta = 90^0$). Effect of orientation of the elliptical cutout in the skew plate is analyzed for five different positions.

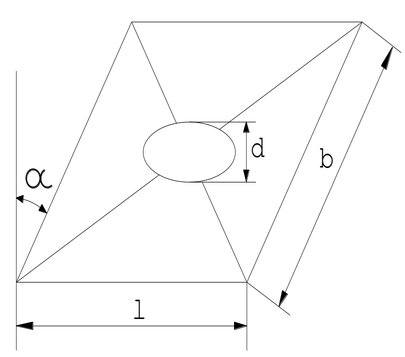

Fig 2. Configuration 1: Major axis of the ellipse parallel to the horizontal side of the skew plate

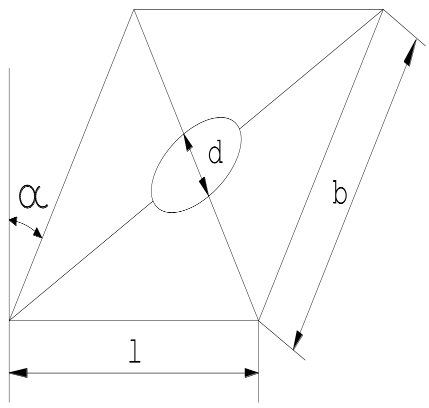

Fig 3. Configuration 2: Major axis of the ellipse collinear with the longer diagonal of the skew plate.

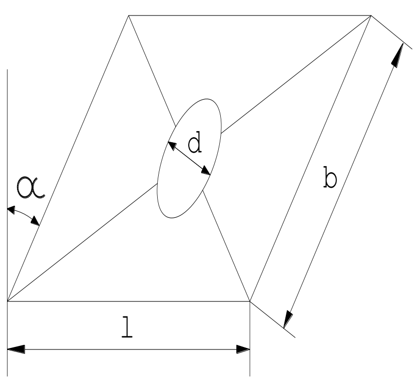

Fig 4. Configuration 3: Major axis of the ellipse parallel to the inclined side of the skew plate.

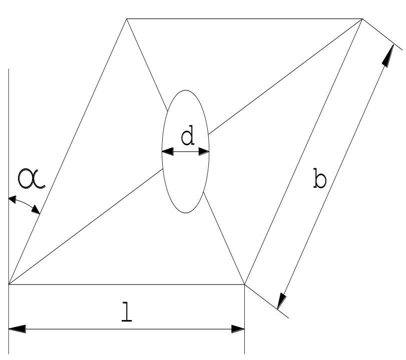

Fig 5. Configuration 4: Major axis of ellipse perpendicular to the horizontal side of the skew plate.

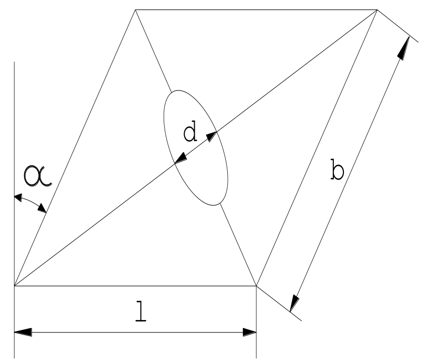

Fig 6. Configuration 5: Major axis of the ellipse collinear with the shorter diagonal of the skew plate.

3.2 Finite element modeling

The finite element mesh is generated using a three-dimensional brick element 'SOLID 95'of ANSYS[11]. This element (Fig.7) is a structural solid element designed based on three dimensional elasticity theory and is used to model thick orthotropic solids. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element may have any spatial orientation.

3.3 Boundary Conditions

All the edges of the skew plate are clamped i.e. all the three degrees of freedom (Displacements in global x-, y- and z- directions) of the nodes attached to the side faces of the plate are constrained

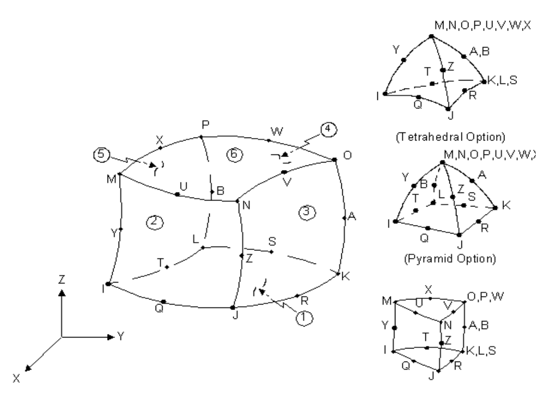

Fig 7. SOLID95 Element

3.4 Loading

All the points of the laminated plate are subjected to uniform temperature rise of 100°C.

3.5 Material Properties (Graphite-Epoxy)

E_1 = 172.72 GPa, $E_2 = E_3$ = 6.909 GPa
$G_{12} = G_{13}$ = 3.45 GPa, G_{23} = 1.38 GPa,
$\nu_{12} = \nu_{13} = \nu_{23}$ = 0.25
$\alpha_L = 0.57 \times 10^{-6}$ /°C $\alpha_T = 35.6 \times 10^{-6}$ /°C

4. VALIDITY OF THE PRESENT ANALYSIS

To validate the finite element results, a square plate with simply supported edges and subjected to a sinusoidal load of $p = p_0 \sin(\pi x/a) \sin(\pi y/b)$, where a and b are the length and width of the plate, is modeled with SOLID95 element. The results obtained from this model are compared with the exact elasticity solution [5] for various lengths to thickness ratios of the plate (Table 1). It is observed that the finite element results are in close agreement with the exact elasticity solution.
Table 1. Presents the results of a square laminate ('a' = 'b'). The location for maximum σ_x i.e. (a/2, a/2, ± 1/2) is at the centre and at top and bottom faces of the laminate. Maximum value of σ_y is observed at the centre and at the interface of outer and its adjacent layers (a/2, a/2, ±1/3) of the laminate. The value of the shear stress τ_{yz} is taken at the mid point of one of the vertical sides and at the neutral surface of the laminate (0, a/2, 0). The value of the shear stress τ_{zx} is taken at the mid point of one of the horizontal sides and at the neutral surface of the laminate (a/2,0,0) and the transverse deflection is obtained at geometric centre of the laminate (a/2,a/2,0).

In the present work the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the elliptical cutout for five different orientations) of a clamped skew cross ply laminated plate with elliptical cutout at the centre of the plate and subjected to a temperature loading is evaluated by varying the size of the elliptical cutout, skew angle and the effect of orientation of elliptical cutout in the skew plate.

Table 1 – Comparison of present work with exact elasticity theory [5]

S = l/h	10	20
Normalized σ_x (a/2,a/2,± 1/2)	EL 0.545 -0.545 FE 0.537 -0.536	EL 0.539 -0.539 FE 0.534 -0.535
Normalized σ_y (a/2,a/2,±1/3	EL 0. 430 -0.432 FE 0.431 -0.431	EL 0.380 -0.380 FE 0.377 -0.378
Normalized τ_{yz} (0,a/2,0)	EL 0.223 FE 0.209	EL 0.212 FE 0.218
Normalized τ_{zx} (a/2,0,0)	EL 0.258 FE 0.212	EL 0.268 FE 0.271
Normalized w (a/2,a/2,0)	EL 0.677 FE 0.692	EL 0.4938 FE 0.4838

5. RESULTS AND DISCUSSION

Numerical results are obtained for temperature loading case as mentioned above. Variation of the stresses and deflection with respect to the skew angle (α), the ratio of length of the minor axis of the elliptical cutout to the side length of the plate (*d/l*) is shown in Fig.8-16. The following observations are made.

5.1 Effect of Skew Angle

The in-plane normal stresses σ_x decrease with the increase in skew angle. The increase in skew angle increases the length of the longer diagonal and decreases the length of the shorter diagonal of the skew plate. The first factor (increase in the length of the longer diagonal) increases the flexibility of the plate where as the second factor (decrease in the length of the shorter diagonal) increases the stiffness of the plate. The reduction in the stress σ_x is due to the domination of stiffness effect.

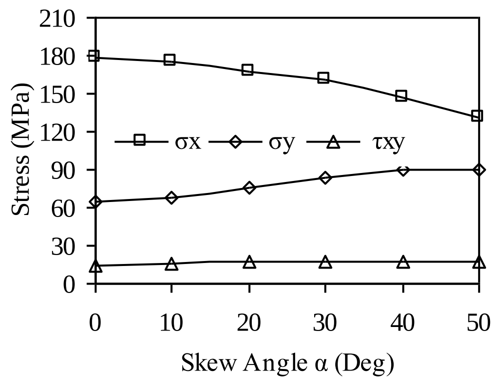

Fig 8: Variation of in-plane stresses with respect to α

The in-plane normal stress σ_y increases up to $\alpha = 40^0$ this is due the domination of first factor and then decreases up to $\alpha = 50^0$ due the domination of second factor with respect to the variation of the skew angle. There is no significant variation in the in-plane shear stress τ_{xy} (Fig8).

The interlaminar stresses at the free edge of the elliptical cutout τ_{yz} increase with increase in skew angle and τ_{zx} increase with increase in skew angle up to $\alpha = 40^0$ and then decreases (Fig. 9). The interlaminar stress σ_z at the free edge of the elliptical cutout decreases with the increase in the skew angle up to $\alpha = 40^0$ and then increases. However the variation is very small when compared to the magnitude of the in-plane stresses.

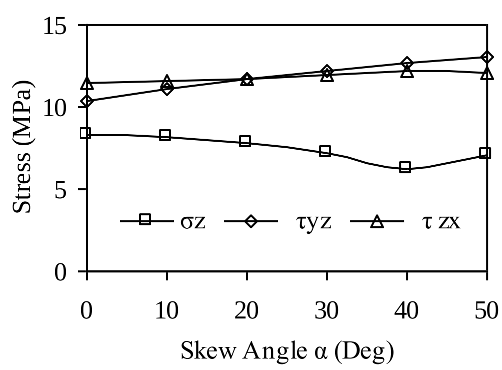

Fig 9. Variation of interlaminar stresses with respect to α

There is no significant variation of transverse deflection '*w*' with respect to α. (Fig. 10).

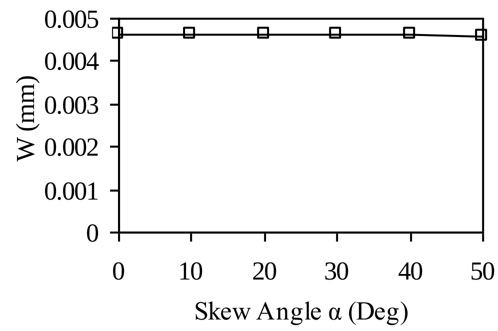

Fig 10. Variation of 'w' with respect to α

5.2 Effect of *d/l* Ratio

When the size of the ellipse increases, the area supporting the load decreases. Due to this the net force acting on the plate decreases causing for the reduction in stresses. At the same time the resisting volume of the material decreases and as a result the induced stresses will increase. The resultant effect of these factors is discussed below.

The in-plane stress σ_x and the in- plane shear stress τ_{xy} increase with increase in *d/l* ratio up to d/l=0.2 and then decreases. The in- plane normal stress σ_y decreases with increase in *d/l* ratio. This is due to the reduction in stress concentration with increase in size of the cutout. (Fig. 11).

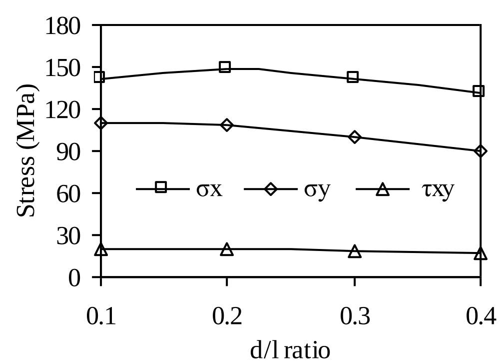

Fig 11: Variation of in-plane stresses with respect to *d/l*

The interlaminar stresses at the free edge of the elliptical cutout σ_z, τ_{yz} and τ_{zx} increase with increase in *d/l* ratio (Fig. 12). The forces causing the interlaminar stresses form in couples to balance the forces for equilibrium. When the size of the cutout increases, the moment arm of these forces decreases and this may be the reason for increase in interlaminar stresses.

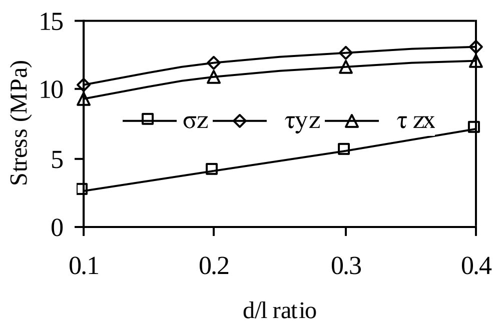

Fig 12. Variation of interlaminar stresses with respect to *d/l*

There is no significant variation transverse deflection '*w*' (Fig. 13).

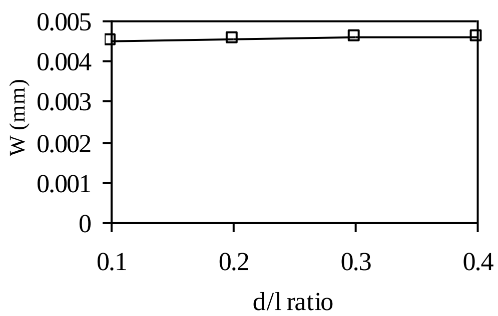

Fig 13. Variation of '*w*' with respect to *d/l*

5.3 Effect of Ellipse Configuration (Orientation)

Five different configurations of the elliptical cutout are analyzed to evaluate the better configuration with minimum magnitudes of major stresses (σ_x and σ_y).

The in-plane normal stresses σ_x and σ_y are minimum at configuration 3. (Fig. 14). There is no significant variation of τ_{xy} with respect to the configuration of the elliptical cutout.

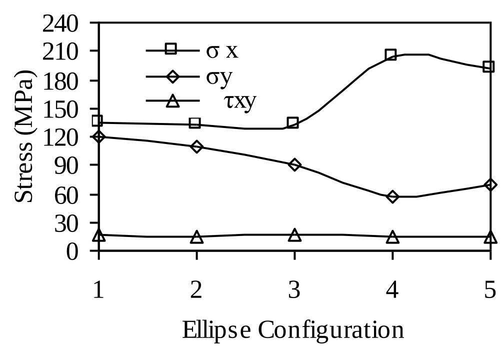

Fig 14: Effect of ellipse configuration on in-plane stresses.

The interlaminar stresses $\sigma_{z,}$ τ_{yz} and τ_{zx} is minimum for configuration 1 (Fig. 15). If the design is based on in-plane strength, configuration 3 is preferred. If the design is based on interlaminar strength configuration 1 is preferred.

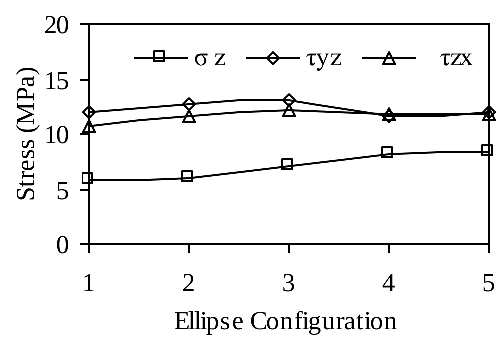

Fig 15: Effect of ellipse configuration on interlaminar stresses.

The transverse deflection '*w'* is minimum for configuration 1 as compared to other configurations (Fig. 16). This configuration may be preferred for the design of the skew plate based on stiffness.

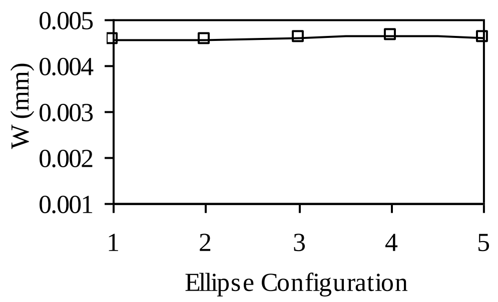

Fig 16: Effect of ellipse configuration on deflection

6. CONCLUSIONS

Thermo elastic analysis of a laminated composite skew plate with an elliptical cutout at the centre of the plate for five different configurations has been carried out in the present work. The transverse deflection, maximum in-plane stresses and maximum interlaminar stresses at the free edge of the cutout have been evaluated using 3-dimensional theory of elasticity based finite element Analysis. The results obtained for uniform transverse pressure loading are analyzed for the variation of skew angle of the plate, size of the ellipse, and configuration of the cutout. The magnitudes of the in-plane normal stresses and the transverse deflection due to temperature loading are greatly affected by the skew angle variation and their magnitudes are observed to be minimum at higher value of the skew angle and *d/l* ratio. Configuration 3
is observed to be better in view of the in-plane normal stresses.

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8. NOMENCLATURE

Symbol	Meaning
E_1	Young's modulus of the lamina in the fibre direction
$E_2 = E_3$	Young's modulus of the lamina in the transverse direction of the fibre
$G_{12}= G_{13}$	Shear modulus in the longitudinal plane of the fibre
G_{23}	Shear modulus in the transverse plane of the fibre
$\nu_{12} = \nu_{13}$	Poisons ratio in the longitudinal plane of the fibre
ν_{23}	Poisons ratio in the transverse plane of the fibre
EL	Exact elasticity solution

FE	Finite element solution
S	Length of the plate (*l*) / Thickness of the plate (*h*)
Normalized **σ** =	$\frac{\sigma}{p_0 s^2}$
Normalized **τ** =	$\frac{\tau}{p_0 s}$
Normalized **w =**	$\frac{100 E_2 w}{p_0 h s^4}$
P_0	The maximum intensity of sinusoidal load
a	length and width of the square plate
$P_{1\text{-}5}$	Orientation of elliptical cutout at the centre of the skew plate
1/2 and 1/3	are the normalized positions along the thickness direction
Normalized *z* =	$2z / h$ *z* coordinate measured from middle plane of the plate and *h*=total thickness of the plate